DIALOG PROGRAM FOR THE TWO-CRITERIAL DYNAMIC LOT SIZE MODEL

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The dynamic lot size model with two objectives is studied. The criterion of minimizing the sum of set-up costs and holding costs is complemented by the minimization of the stock. Solutions which are efficient with respect to these two objectives can be derived from parametric one-criterial models with combined objective functions. A complete set of efficient solutions which are distinct in their objectives can be found by a BASIC dialog program for personal computers.

1. THE PROBLEM

The one-criterial problem (Wagner 1969) can be introduced as follows: The process of production and stock-holding for one item and T periods is considered. The production figures $x_t \geqslant 0$ for t^{th} period, $t = 1, 2, \ldots, T$, have to be chosen such that given deterministic demand $d_t \geqslant 0$ is satisfied for all t and that the total sum of set-up costs and holding costs is minimal. The fixed set-up costs arising if $x_t > 0$ are denoted by $c > 0$ and the per-unit period linear holding costs are denoted by $h > 0$. If the stock at the end of t^{th} period is denoted by y_t the demand is satisfied in the case that $y_t = y_{t-1} + x_t - d_t$ is non-negative. Usually the assumption is made that the stock equals zero at the beginning and at the end of the considered planning period $1, 2, \ldots, T$, i.e.

$$y_0 = y_T = 0.$$

Then the model can be described by the formulae (1)–(4):

$$y_t = y_{t-1} + x_t - d_t \tag{1}$$

$$t = 1, 2, \ldots, T,$$

$$x_t \geqslant 0, \; y_t \geqslant 0, \tag{2}$$

$$y_0 = y_T = 0, \tag{3}$$

$$F_1 = c \sum_{t=1}^{T} \operatorname{sign} x_t + h \sum_{t=1}^{T} y_t \rightarrow \min \tag{4}$$

The second criterion consisting in the minimization of the stock is provided by formula (5):

$$F_2 = \sum_{t=1}^{T} y_t \rightarrow \min \tag{5}$$

The objective F_2 makes sense only if the first criterion is also of interest, since the solution $x = \{x_t = d_t\} \sum_{t=1}^{T}$ provides always $F_2 = 0$.

The one-criterial problem will be denoted by $M(c, h)$. Feasible solutions of this model have to satisfy conditions (1)–(3), while optimal solutions are feasible solutions minimizing the function (4).

Let two feasible solutions be considered which have the values F_1', F_2' and F_1'', F_2'', respectively. The first solution is said to be dominated by the second one if $F_1' \geqslant F_1''$ and $F_2' \geqslant F_2''$. It is strongly dominated if the inequalities hold and one of them is strong. An efficient solution can be defined as a feasible solution which is not strongly dominated by any other feasible solution. In other words, the feasible solution associated with the values F_1', F_2' is efficient if it follows for any other feasible solution that

$F_1'' < F_1'$ implies $F_2' < F_2''$ and

$F_2'' < F_2'$ implies $F_1' < F_1''$.

The main task in multi-criterial optimization is to find set EFF of all efficient solutions of a given problem. In many cases it is preferable to characterize a subset EFF' of EFF which covers all possible pairs of values of EFF, i.e. EFF' contains at least one efficient solution for each pair of values. The aim of the paper is to describe such subset for model (1)–(5) and to illustrate how these solutions can be found by a computer program.

Example: Let $T=3$, $c=5$, $h=2$, $d_1=3$, $d_2=2$, $d_3=1$.

The optimal solution $(x, y) = \{x_1=3,\ x_2=3,\ x_3=0,\ y_1=0,\ y_2=1,\ y_3=0\}$ of problem (1)–(4) with $F_1=12$ and $F_2=1$ is an efficient solution as it will be shown later on.

2. STABILITY AND MONOTONY OF THE ONE-CRITERIAL MODEL M(c, h)

The optimal solutions can be found using the following recursive procedure (comp. Richter 1982):

Let

$$T_0 = \{t: d_t > 0\},\ h(k, l) = h \sum_{t=k+1}^{l} (t-k-1)d_t$$

and

$c(k, l) = c + h(k, l)$ for $k<l$ and $d_{k+1}>0$.

Then

$$f_0 := 0, f_t := \min\{c(k, t)+f_k : k \leqslant t-1,\ k \in T_0 - \{1\}\} \tag{6}$$

for all $t \geqslant 1$ and $t \in (T_0 - \{1\}) \cup \{T\}$ can be used to determine the minimal value $F_1 = f_T$ for $M(c, h)$.

Let

$$f(k, t) = c(k, t) + f_k$$

and let the parameters $k(t)$ be introduced by

$$f_t = f(k(t), t) \tag{7}$$

for all suitable t. Then an optimal solution can be found using these parameters.

Algorithm: Input $k(t)$ for $t \in (T_0 - \{1\}) \cup \{T\}$.

1. $t := T$.
2. $x_{k(t)+1} := \sum_{r=k(t)+1}^{t} d_t$, $x_r := 0$ for $r = k(t)+2, \ldots, t$.
3. If $k(t) = 0$ stop, else $t := k(t)$, go to step 2. Output $x_1, x_2, \ldots, x_T$.

The components of the vector y can be determined by formula (1). Since the parameters $k(t)$ play the most important role in solving the one-criterial problem the collection

$$K = \{k(t)\}_{k \in (T_0 - \{1\}) \cup \{T\}} \tag{8}$$

is called generalized solution. The stability of model (1)–(4) can be studied easily in terms of generalized solutions.

Let K be a generalized solution and let the following parameters be introduced:

(i) $\bar{c}(0) := 0$, $\bar{c}(t) := \bar{c}(k(t))+1$ for all suitable t.
(ii) $r(k, t) = (f(k, t) - f_t)/(\bar{c}(k(t)) - \bar{c}(k))$ for all suitable k and t.
(iii) $\text{low} = \max_t \max_k \{r(k, t): \bar{c}(k(t)) < \bar{c}(k)\}$ and $\text{up} = \min_t \min_k \{r(k, t): \bar{c}(k(t)) > \bar{c}(k)\}$,

where low and up can be set minus or plus infinity, respectively, if they are not defined.

The following stability region can be found for a given generalized solution.

Theorem 1 (Richter 1984): K is a generalized solution for all $M(c', h')$ with $c', h' > 0$ and

$$(c+\text{low})/h \leqslant c'/h' \leqslant (c+\text{up})/h$$

and it is not a generalized solution for any other pair $(c', h') > 0$.

Let the example from section 1 be studied. We find the generalized solution $K = (0, 0, 1)$ and the optimal solution mentioned there. The calculations are presented in Table 1.

TABLE 1

Solving the problem

		$f(k, t)$			$r(k, t)$		
k	t	1	2	3	1	2	3
0		5	9	13	—	—	1
1		—	10	12	—	−1	—
2		—	—	14	—	—	—
f_t		5	9	12			
$k(t)$		0	0	1			
$\bar{c}(t)$		1	1	2			

We find low $= -1$, up $= 1$ and see that the stability region is given by $2 \leqslant c'/h' \leqslant 3$.

Theorem 2 (Richter 1984): Let $d_1, d_2, \ldots, d_T$ be a fixed sequence of demand values. Then there is a finite number of generalized solutions and associated stability regions defined by the inequalities in Theorem 1 which cover R^2_+.

One can see the four stability regions and generalized solutions of $d_1 = 3$, $d_2 = 2$, $d_3 = 1$ in Fig. 1.

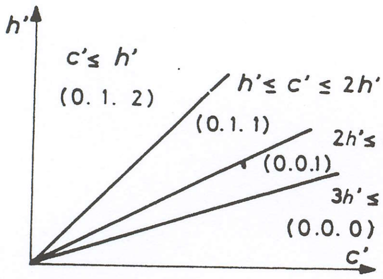

Fig. 1. Stability regions

Since the optimal solution can be also derived from the generalized solution $K = (0, 1, 1)$ the stability region of the found optimal solution covers that of (0, 0, 1) and (0, 1, 1).

We shall provide here only one of the possible monotony results for optimal solutions.

Theorem 3 (Richter 1986): Let two pairs of cost inputs $0 < c'/h' < c''/h''$ be given and let (x', y') and (x'', y'') be the corresponding optimal solutions. Then the following inequalities hold:

$$\text{(i)} \ \sum_{t=1}^{T} y'_t \leqslant \sum_{t=1}^{T} y''_t$$

$$\text{(ii)} \ \sum_{t=1}^{T} \operatorname{sign} x'_t \geqslant \sum_{t=1}^{T} \operatorname{sign} x''_t$$

A BASIC dialog program has been designed to carry out a stability analysis for a given problem $M(c, h)$. By investigating the neighbouring stability regions one can find not only those regions for generalized but also for optimal solutions of the problem (1)–(5).

3. COMPLETE CHARACTERIZATION OF EFF'

We consider the parametric model

$$M(ac, ah + 1 - a) \qquad (9)$$

for $0 \leqslant a \leqslant 1$. Then the following statement is true.

Theorem 4 (Richter 1986): All generalized (and therefore also optimal) solutions of the problem (9) for $0 < a < 1$ are efficient solutions for the model (1)–(5). All other feasible solutions of (1)–(5) are dominated by one of the found efficient solutions.

It follows from this theorem that the generalized solutions provide an efficient solution, if the line

$$\{c' = ac,\ h' = ah + 1 - a\colon 0 < a < 1\}$$

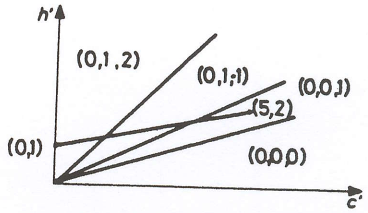

Fig. 2. Stability regions of the efficient solutions

touches their stability region. One can see in Fig. 2 that only three generalized solutions provide efficient solutions for our example.

TABLE 2

Results of the dialog

No. of solution	1	2	3	4	5	Demand
F_1	2600	3300	4150	5050	6000	
$F_{2'}$	120	60	30	10	0	
lower bound	.078	.034	.021	.01	0	
upper bound	1	.078	.034	.021	.01	
x_1	130	100	100	100	100	100
x_2	0	0	30	20	20	20
x_3	0	0	0	0	0	0
x_4	0	0	0	10	10	10
x_5	60	40	40	40	30	30
x_6	0	0	0	0	10	10
x_7	0	0	0	0	0	0
x_8	0	20	20	20	20	20

$c = 1000$
$h = 5$

The BASIC dialog program is designed to use the mentioned results in the following way. First, an optimal solution is generated together with the lower and upper bounds for the parameter a. By choosing other parameters a outside these bounds the decision maker can try to find other efficient solutions which fit better his conception on the relationship between F_1 and F_2.

In Fig. 2 the results of the dialog with a problem of eight periods are provided.

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