A General Production and Recovery EOQ Model with Stationary Demand and Return Rates

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This paper considers the general production and recovery EOQ model (GPRM), which generalizes the model of Saadany and Jaber (2008). In this general model, one supplier and one or several buyers constitute the underlying supply chain. The supplier is supposed to manufacture new products, which are then delivered to the buyers according to fixed demand rates. The supplier is also capable of recovering used products (cores),which are returned back by the buyers. Our modelling approach generalizes a whole class of various other models that draw attention to different aspects of production, inventory, and recovery. A complete solution in the form of a theorem for that general model class is provided. Furthermore, the paper illustrates how that theorem can be applied to one of the mentioned models from the literature.

1 Introduction

In recent years, reverse logistics has received increasing attention from both academia and industry. There is increasing recognition that careful management can bring both environmental protection and lower costs: environmental and economic considerations have led to manufacturers taking their products back at the end of their lifetimes. As a result, the reverse logistics process is now considered as a basis for generating real economic value as well as supporting environmental concerns.

Rogers and Tibben-Lembke [24] defined reverse logistics as the process of planning, implementing, and controlling the efficient, cost-effective flow of raw materials, in-process inventory, finished goods, and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal. The integration of forward and reverse supply chains resulted in the origination of the concept of a closed-loop supply chain. The whole chain can be designed in such a way that it can service both forward and reverse processes efficiently.

One of the most recent full reviews of quantitative modelling for inventory and production planning in a closed-loop supply chain was published by Akcaly and Cetinkaya [1]. Inventory models are divided into two main categories: deterministic and stochastic, according to the modelling of demand and return processes. The subject of this paper is deterministic inventory models with constant demand and return.

The economic order quantity (EOQ) model, which was derived by Ford W. Harris in 1913, became the basis for many reverse logistics models because of its simplicity and intelligibility. Paper [2] represents the most detailed review devoted to the work on the EOQ problem.

As an example, we consider an EOQ repair and waste disposal model that was introduced by Richter in 1996 [20]. A first shop is providing a homogeneous product used by a second shop at a constant demand rate of d items per time unit. The first shop is manufacturing new products and is also repairing products used by a second shop, which are then regarded as being as good as new. The products are employed by a second shop and collected there according to a repair rate β. The other products are immediately disposed of as waste according to the waste disposal rate $\alpha = 1-\beta$. At the end of some period of time $[0,T]$, the collected products are brought back to the first shop and will be stored for as long as necessary and then repaired. If the repaired products have all been sold, the manufacturing process starts to cover the remaining demand for the time interval. There are three inventories in this model: NII, the manufactured and remanufactured item inventory (or new item inventory) and UII1 and UII2, the used item inventories for the first and second shops. The model is depicted in Fig. 1.

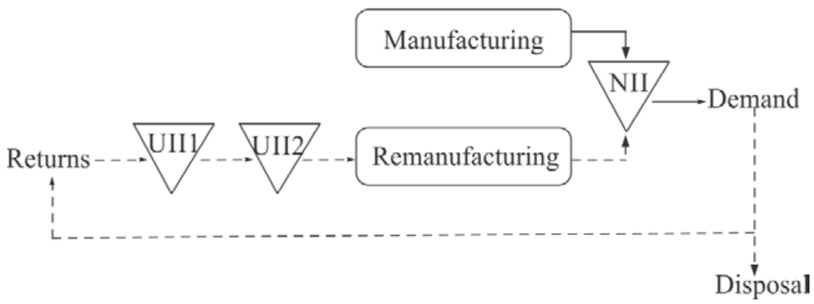

Figure 1: Inventory system in EOQ repair and waste disposal model

Richter [21],[22] further assumed that each time interval starts with repair runs (lots), where these runs (lots) are followed by manufacturing runs (lots). In the paper by Saadany and Jaber [25], the extended EOQ production, repair, and waste disposal model [20] was modified to show that ignoring the first time interval results in an unnecessary residual inventory and consequently an over-estimation of the holding costs. The dynamics of inventories are illustrated in Fig. 2. The UIIs of the first and second shops are considered together.

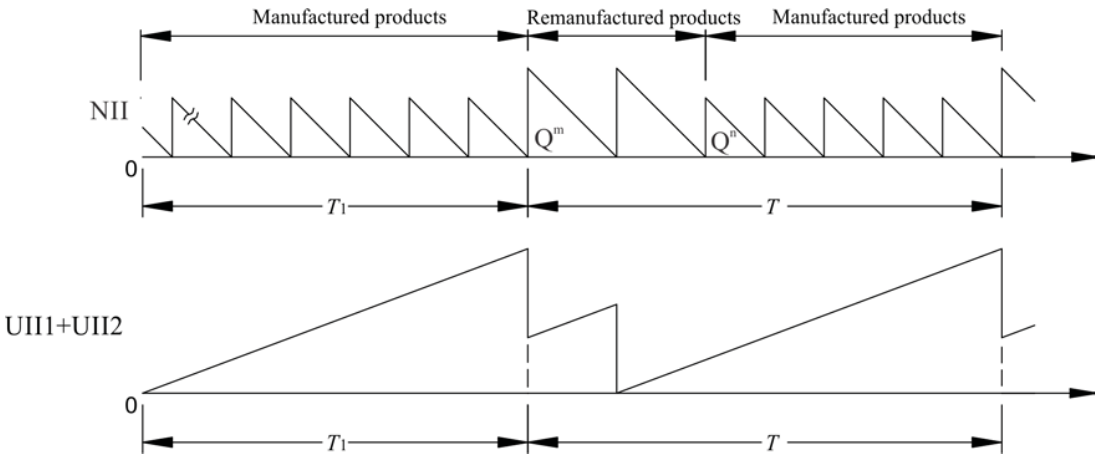

Figure 2: The modified behaviour of NII and UII

The main peculiarity of the paper [25] is that it accounts for switching costs (e.g., production loss, deterioration in quality, additional labour). When shifting from producing (performing) one product (job) to another in the same facility, the facility may incur additional costs referred to as switching costs. In this model, two processes are modelled: manufacturing and remanufacturing. Let n be the number of newly manufactured lots in an interval of length T, m the number of remanufacturing lots in an interval of length T, d the demand rate (units per unit of time), and Q^n and Q^m the size of manufacturing and remanufacturing lots, respectively:

$$nQ^n = \alpha dT,$$
$$mQ^m = \beta dT.$$

Then

$$Q^n = \frac{\alpha dT}{n},$$
$$Q^m = \frac{\beta dT}{m}.$$

The cost function is the sum of setup costs, switching costs, and inventory holding costs. Setup costs depend on the numbers of lots: $(m-1)r+(n-1)s$; switching costs depend on the production scheduling (two switches between manufacturing and remanufacturing processes in an interval of length T): r_1+s_1. Inventory holding costs are the sum of holding costs at stock points, namely holding costs for used items and holding costs for manufactured and remanufactured items, which depend on dynamics of inventories, lot sizes, and numbers of lots (see Fig. 2).

According to Saadany and Jaber (2008) [25], the modified cost function per time unit in the model of Richter (1996) [20] with switching costs is equal to

$$K_2(Q^m, Q^n, m, n, \alpha, T) = \frac{(m-1)r + r_1 + (n-1)s + s_1}{T} + \\ + \frac{h}{2dT}\left(n(Q^n)^2 + m(Q^m)^2\right) + \frac{u\beta dT}{2} - \frac{u\beta^2 dT(m-1)}{2m}. \tag{1}$$

Excluding Q^n, Q^m (1) and differentiating with respect to T, we obtain:

$$K(m,n) = \sqrt{2d(r_1 - r + s_1 - s + mr + ns)(u\alpha\beta + (h+u)\beta^2 \frac{1}{m} + h\alpha^2 \frac{1}{n})}. \quad (2)$$

In [25], however, the authors did not provide a complete solution to this complex problem. The special case of even numbers m and n was studied and conditions were provided to decide which of two policies is preferable, but a general optimal policy for the problem was not presented.

Our paper develops a general class of deterministic multi-product inventory models with constant demand and return that generalizes the approach of Saadany and Jaber and contains other published types of models. In all these models, three types of costs are considered. First, the EOQ-unrelated cost, which is independent of the numbers of lots and lot sizes (the production cost), the EOQ-related cost that depends on the dynamics of the inventory, the lot sizes, and numbers of lots (the holding cost), and the EOQ-related cost that depends on the numbers of lots and production scheduling (the switching cost). In this paper, a complete solution to this class of models is provided. Furthermore, the paper also illustrates how various other remanufacturing problems can be solved by specifying this solution.

In our paper, we present a method of finding the optimal solution for this concrete model and some other models. This paper represents an attempt to generalize some of the already existing models and to provide the common methods used to solve them. In the paper [39], the problem of Saadany and Jaber (2008) [25] was solved by using the abovementioned approach for the case of a single product.

2 Review of the literature

As there is a wide variety of deterministic inventory models with constant demand and return on the basis of EOQ, let us formulate the following common conditions of the models considered in order to narrow the scope:

(1) production and recovery rates are deterministic, constant, or infinite
(2) there is an infinite planning horizon
(3) disposal, return, and other rates are deterministic and constant
(4) setup, switching, and holding costs are known
(5) product demand rates are deterministic and constant;
(6) only one product can be produced at a time on the same production line
(7) production scheduling and inventory control strategy are predetermined

We focus on the following differences in modelling: types of logistics procesess considered, types of costs, and number of stock points. Processes in the supply chain are subdivided into reverse and direct processes. Reverse logistics activities are discussed in paper [34], and mainly include all forms of recovery: direct reuse, repair, refurbishing, cannibalization, recycling, incineration, and landfilling. Recovery is actually only one of the activities involved in the whole reverse logistics process. Collection takes place first, followed by the combined inspection/selection/sorting process and then recovery (which may be direct or may involve a form of reprocessing), and finally redistribution. Collection refers to bringing the products from the customer to a point of recovery. At this point the products are inspected, that is, their quality is assessed and a decision is made on the type of recovery. Products can then be sorted and routed according to the recovery that follows. If the quality is (close to) "as good as new", products can be fed into the market almost immediately through reuse, resale, and redistribution. If not, another type of recovery may be involved, but requires more action, that is, a form of reprocessing.

Reprocessing can occur at different levels: product level (repair), module level (refurbishing), component level (remanufacturing), selective part level (retrieval), material level (recycling), and energy level (incineration).

Also we consider other activities involved in the whole reverse logistics process: collection, sorting, inspection [16], dismantling into components, assembly of components [36], and so on, which can result in additional costs. All authors consider two types of costs: setup costs and holding costs, but different authors also consider a number of other costs that make up the total costs: raw materials cost, purchase cost [36], manufacturing cost [16], remanufacturing cost [16], refurbishing cost [16], disposal cost [6],[7], [9], [20],[22], repair cost, switching [25], changeover cost, dismantling operation cost, lost sale cost [10], backordering cost [9],[10], and inspection cost [16]. In the paper [1], the following systems are distinguished considering stock points for manufactured item inventory (MII), manufactured and remanufactured item inventory (MRII), new material inventory (NMI), remanufactured item inventory (RII), and used item inventory (UII). There are systems with one, two, and multiple stock points and systems with one product and more than one product (see table 1.).

Number of Stock Points	Authors
2	Shrady (1967) [28]; Nahmias and Rivera (1979) [19]; Mabini et al. [18]; Teunter (2001) [30]; Teunter (2004) [31]; Dobos and Richter (2004) [7]; Konstantaras and Papachristos (2006) [13]; Konstantaras and Papachristos (2008) [14]; Konstantaras, Scouri, and Jaber (2010) [15]; Hasanov, Jaber, and Zolfaghari (2012) [9]
3	Richter (1996) [20], Richter (1997) [22]; Saadany and Jaber (2008) [25]
> 3	Chung, Wee, and Yang (2008) [38], Pishchulov, Dobos, Gobsch, Pakhomova, and Richter (2014) [37]

Table 1: Inventory systems with two, thee, and more stock points.

3 Formulation of the general model

The supply chain consists of a supplier and a buyer or buyers. The supplier can produce new products or recover cores, which are returned by the buyer or buyers. The framework of the inventory system is presented in Fig. 3. Activities in the supply chain are classified as related to reverse logistics (recovery) activities and to direct (production) activities. Manufacturing of new products refers to production activities. Reverse logistics activities are discussed in paper [34] and in Section 2. Repair and remanufacturing refer to reverse logistics. The goal of repair is to restore failed products to working order, although possibly with a loss of quality [42]. In the case of remanufacturing, products are dismantled and used and new parts can be used in the manufacturing of either the same products or different ones. It is supposed that after the product is returned, it is inspected and found to be either repairable or not repairable and then sorted; in the first case the repair process starts; in the second case, remanufacturing starts. It is supposed that the repair cost is less the cost of remanufacturing and is preferable. For simplicity, it is supposed that recovered products are as good as new. In this model two activities that refer to reverse logistics are considered, but the results obtained can be generalized for many activities.

Let us denote the demand for product by d. Let T be the time length of the cycle; then dT

is the demand for the product that occurs per time interval *[0,T]*. This demand is served by manufacture of new items as well as by recovering some part.

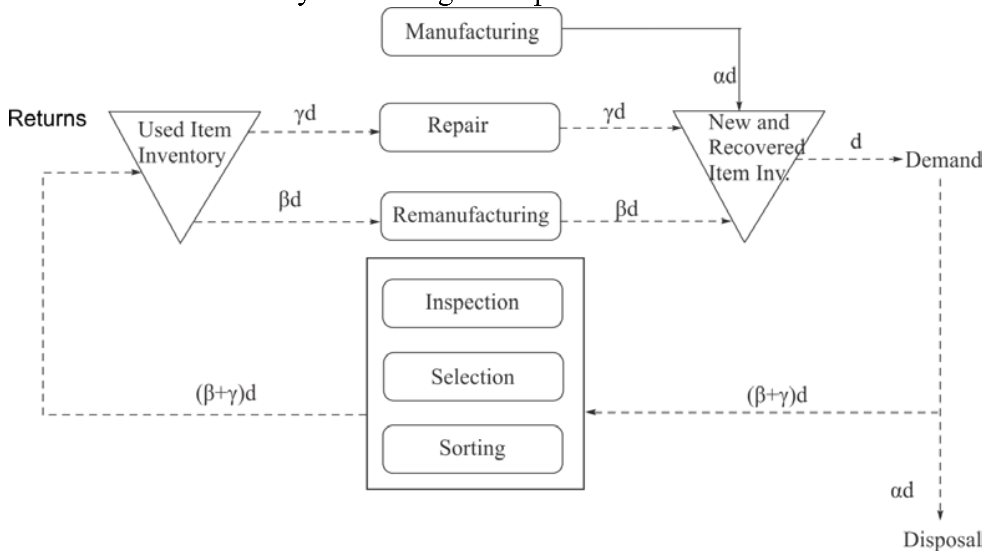

Figure 3: Inventory system in the general model

All production and recovery activities are implemented on the same production line. The sequence of production and recovery activities is given and does not change from cycle to cycle. It becomes the same except for the first time interval (because recovery activities are not available, when used item inventory is empty). Under the production scheduling, we understand the sequence of production and recovery activities. The production scheduling is supposed to be predetermined. In this model, repair precedes remanufacturing, and remanufacturing precedes manufacturing.

It is supposed that the production and recovery rates are deterministic and constant. In some models, the production and recovery activities are supposed to be instantaneous, which is modelled by infinite production and recovery rates [5,6,7,22,23,25,27]. Let us denote the manufacturing as *p*, the remanufacturing rate as r_1, and the repair rate as r_2. It is supposed that $p > d, r_1 > d$, and $r_2 > d$.

Newly produced products, used products, and materials are collected in stock points. The production scheduling and control strategy determine the dynamics of inventory in stock points. The production or recovery process can be instantaneous or can take time [11,14,16,19], and hence the inventory level in the stock point is changed by the lot size either instantaneously or gradually over time at a constant rate. In this model, two stock points are considered: new and recovered item inventory with *H* the holding cost per item per time unit, and used item inventory with *h* the holding cost per item per time unit; $H > h$. The dynamics of inventories are represented in Fig. 4.

The dynamics of inventory in a stock point depends on the inventory control strategy. In the literature, two control strategies are mentioned: the push control strategy (in which all returned products are recovered as early as possible) and the pull control strategy (in which all returned products are recovered as late as is convenient), and different control strategies lead to different dynamics of inventory even under the same production scheduling (see Section 6.).

To determine the optimal policy under a predetermined production scheduling and control strategy, the lot sizes and optimal numbers of lots must be found. Here we consider one type of production activity – manufacturing of new product – and two types of reverse activity – remanufacturing and repair. Later this case may be generalized for many reverse and direct activities.

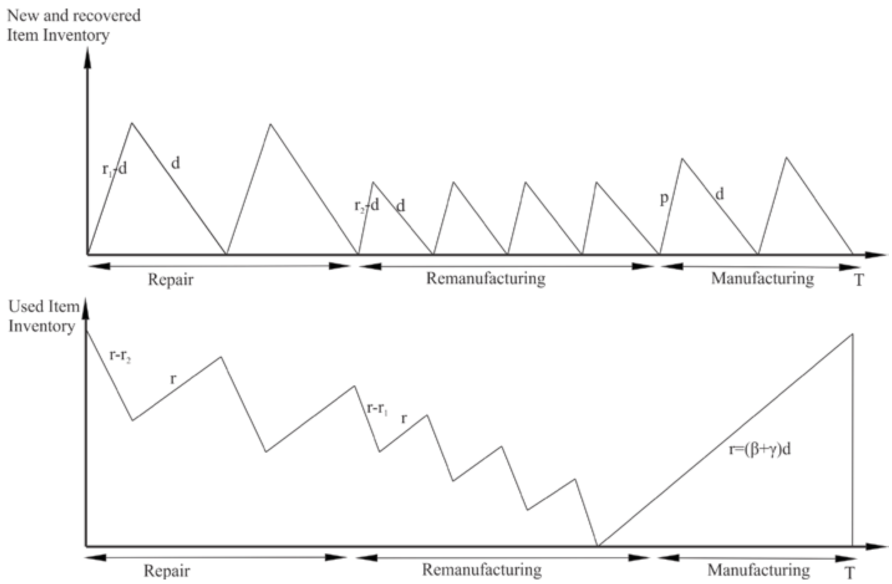

Fig. 4. Dynamics of inventories.

Denote by α the share of manufactured product.

Denote by β, γ the share of product which is reprocessed using reverse logistics activities: let β be the share of remanufactured product and γ the share of repaired product.

The following condition holds:

$$\alpha + \beta + \gamma = 1.$$

This paper assumes that demand is supplied by dT of product per time interval $[0, T]$. The quantity of dT is obtained through αdT of manufactured items in n lots of size Q_n, βdT of remanufactured items in m lots of size Q_m, and γdT of repaired items in k lots of size Q_k. We have the following system of equations:

$$\begin{aligned} \alpha dT &= nQ_n \\ \beta dT &= mQ_m \\ \gamma dT &= kQ_k \\ \alpha + \beta + \gamma &= 1 \end{aligned} \tag{3}$$

We divide all costs of the supplier into three groups:

1. EOQ-unrelated cost, which does not depend on the numbers of lots and lot sizes at all, that is, production cost, waste disposal cost, repair cost [20], purchase cost, remanufacturing cost, inspection cost, refurbishing cost [16], and so on. It is assumed that all EOQ-unrelated costs in the model are proportional to the quantity or product.
2. EOQ-related cost that depends on the dynamics of the inventories, lot sizes, and numbers of lots, that is, holding cost, back-ordering costs, and lost sales.
3. EOQ-related cost that depends on the numbers of lots and production scheduling, that is, setup cost, order cost, changeover cost, and other switching costs [25].

Let us denote the total cost over $[0, T]$ by

$$TC = TC_I(T, n, m, k, Q_n, Q_m, Q_k). \tag{4}$$

Note that the control strategy and production scheduling, which define the dynamics of the inventory, also define the structure of function (4), so the index I means that total costs are calculated assuming the dynamics of inventory I.

Taking into account system (3), the variables Q_m Q_n, and Q_k can be derived:

$$Q_n = \frac{\alpha dT}{n},$$

$$Q_m = \frac{\beta dT}{m},$$

$$Q_k = \frac{\gamma dT}{k}.$$

and excluded from (4):

$$TC = TC_I(T, n, m, k), \tag{5}$$

Recall that the costs in the model are divided into three types, and also the total cost function over $[0, T]$ is the sum of three terms:

$$TC = TC_I(T, n, m, k) = F(T) + G(n, m, k) + H_I(T, n, m, k), \tag{6}$$

where $F(T)$ is the EOQ-unrelated cost, which does not depend on the numbers of lots and lot sizes at all. We assume that $F(T) = F \cdot T$, where F is a positive constant. This can be easily explained: dT is the quantity of product which is demanded per [*0,T*], αdT is the quantity of products newly produced by production activity *i*, and the production cost will be proportional to the quantity of manufactured products and as a result proportional to T and therefore other EOQ-unrelated costs.

Let $G(n, m, k)$ be the EOQ-related cost, which depends on the numbers of lots and production scheduling and does not depend on the cycle length T:

$$\mathrm{G}(n, m, k) = W + nS + mR + kP, \tag{7}$$

where *W* is the total switching costs that are incurred when the activity is switched from remanufacturing to manufacturing, from repairing to remanufacturing, or from manufacturing to repairing; *S* is the setup manufacturing cost; *R* is the setup remanufacturing cost; and *P* is the setup repair cost.

$H_I(T, n, m, k)$ is the EOQ-related cost, which depends on the dynamics of inventory I, lot sizes (which are excluded using (3)), and numbers of lots, that is, holding cost and backordering cost.

Due to (3), the size of the holding cost H_I depends quadratically on the length of the corresponding cycle T for a given sequence of production and recovery lots. The predetermined scheduling of remanufacturing and manufacturing lots in a cycle does not depend on its overall length. For instance, if T is doubled, all lot sizes are doubled too. Hence the time taken to collect appropriate returns doubles, as does the time during which a (re)manufacturing lot is able to satisfy customer demand. Thus H_I will be four times its initial value if T is doubled. We assume that

$$H_I(T, n, m, k) = T^2 H_I(n, m, k).$$

We have

$$TC_I(T, n, m, k) = F \cdot T + G(n, m, k) + T^2 H_I(n, m, k),$$

It is supposed that $F, G(\cdot), H(\cdot) > 0$.

The unit time cost function is obtained by dividing by T:

$$ATC_I(T, n, m, \mathrm{k}) = F + \frac{G(n, m, \mathrm{k})}{T} + T \cdot H_I(n, m, \mathrm{k}), \tag{8}$$

It can be easily found from (8) that the length of the optimal time cycle is equal to

$$T = \sqrt{\frac{G(n,m,\mathrm{k})}{H_I(n,m,\mathrm{k})}} \tag{9}$$

and the corresponding cost equals

$$ATC_I(n,m,k) = F + 2\sqrt{G(n,m,k)\cdot H_I(n,m,k)}\ , \tag{10}$$

where $G(\mathrm{n,m,k})$ is defined by (7) and $H_I(n,m,k)$ equals:

$$H_I(n,m,k) = H\Big(\frac{1}{2}\frac{d(r_2-d)\gamma^2}{r_2}\frac{1}{k}+\frac{1}{2}\frac{d(r_1-d)\beta^2}{r_1}\frac{1}{m}+\frac{1}{2}\frac{d(p-d)\alpha^2}{p}\frac{1}{n}\Big)+$$

$$+h\Big(\frac{1}{2}\alpha(\beta+\gamma)d+\frac{\beta(\beta+2\gamma)}{2}\frac{d(r_1-d)}{r_1}\frac{1}{m}-\frac{1}{2}\frac{d(r_2-d)\gamma^2}{r_2}\frac{1}{k}\Big)$$

The computations are presented in the Appendix. The function $H_I(n,m,k)$ can be represented in the form:

$$H_I(n,m,k) = a_0+\frac{a_1}{n}+\frac{a_2}{m}+\frac{a_3}{k},$$

$$a_0 = \frac{1}{2}h\alpha(\beta+\gamma)d$$

$$a_1 = \frac{1}{2}H\frac{d(p-d)\alpha^2}{p}$$

$$a_2 = \frac{1}{2}H\frac{d(r_1-d)\beta^2}{r_1}+h\frac{\beta(\beta+2\gamma)}{2}\frac{d(r_1-d)}{r_1}$$

$$a_3 = (H-h)\frac{1}{2}\frac{d(r_2-d)\gamma^2}{r_2}$$

It can be verified that $H_I(n,m,k) > 0$.

The average total costs equals:

$$ATC_I(n,m,\mathrm{k}) = F + 2\sqrt{G(n,m,k)\cdot H_I(n,m,k)} \tag{11}$$

We denote

$$L(n,m,k) = G(n,m,k)\cdot H_I(n,m,k).$$

The aim is to determine the optimal policy, in other words the optimal numbers *n, m,* and *k,* that minimizes the average total cost:

$$\min_{(n,m,k)} ATC_I(n,m,k) = \min_{(n,m,k)}(F+2\sqrt{L(n,m,k)}),$$
$$\mathrm{n,m,k} \in \{1,2,\ldots\} \tag{12}$$

The problem (12) is named the general production and recovery model (GPRM).

Instead of solving the problem (12), the function *L(n,m,k)* can be minimized subject to $n,m,k \in \{1,2,\ldots\}$; that is, the following two-dimensional nonlinear integer optimization problem is relevant:

$$\min_{(n,m,k)} L(n,m,k) = \min_{(n,m,k)}(W+nS+mR+kP)\cdot\Big(a_0+\frac{a_1}{n}+\frac{a_2}{m}+\frac{a_3}{k}\Big),$$
$$n,m,k \in \{1,2,\ldots\}. \tag{13}$$

4 Solution of the general production and recovery model

For the solution of the problem (13), consider the following two-dimensional nonlinear integer optimization problem:

$$\min_{(x_1,x_2,\ldots,x_n)} K(x_1,x_2,\ldots,x_n) = \min_{(x_1,x_2,\ldots,x_n)} (b_0 + \sum_{i=1}^{i} b_i x_i)\cdot(a_0 + \sum_{i=1}^{n}\frac{a_i}{x_i}),$$
$$x_i \in \{1,2,\ldots\},\ \mathrm{i} = 1,2,\ldots n. \tag{14}$$

First, let us consider the following continuous auxiliary problem:

$$\min_{(x_1,x_2,\ldots,x_n)} K(x_1,x_2,\ldots,x_n) = \min_{(x_1,x_2,\ldots,x_n)} (b_0 + \sum_{i=1}^{i} b_i x_i)\cdot(a_0 + \sum_{i=1}^{n}\frac{a_i}{x_i}),$$
$$x_i \geq 1, i = 1,2,\ldots,n. \tag{15}$$

By analysing the first partial derivatives, we can prove the following lemma:

Lemma. *If* $x_i > 0, i = 1,2,\ldots,n$*, there are n curves of local minima (15) with respect to* x_j:

$$X_j(x_1,x_2,\ldots,x_{j-1},x_{j+1},\ldots,x_n) = \sqrt{\frac{a_j(b_0 + \sum\limits_{i=1,i\neq j}^{n} b_i x_i)}{b_j(a_0 + \sum\limits_{i=1,i\neq j}^{n}\frac{a_i}{x_i})}}, \tag{16}$$

and the point of the local minimum

$$x_j^* = \sqrt{\frac{a_j b_0}{a_0 b_j}},\ i = 1,2,\ldots,n. \tag{18}$$

Let us denote the radicands of the expressions (18) by

$$A_i = \frac{a_i b_0}{a_0 b_i},\ i = 1,2,\ldots,n. \tag{19}$$

Without loss of generality, it is supposed that $A_1 < A_2 < \ldots < A_n$.

We denote:

$$B_i(j) = \frac{a_i(b_0 + \sum_{k=1}^{j} b_k)}{b_i(a_0 + \sum_{k=1}^{j} a_k)},\ i = 1,2,\ldots,n. \tag{20}$$

Then the optimal solution for the continuous problem (15) is provided by the following theorem.

Theorem. *The optimal solution to the problem (15) has the following structure depending on the* *value of the parameters* $A_i, B_i(j)$:

1. If $A_i \geq 1, i = 1, 2, \ldots, n$, then $x_i = \sqrt{A_i}$, $i = 1, 2, \ldots, n.$
2. If $A_1 < 1$, then consider $B_2(1), B_3(2), \ldots\ldots, B_j(\text{j}-1), \ldots, B_n(n-1)$; if $B_j(\text{j}-1) < 1$ and $B_{j+1}(\text{j}) \geq 1$ then $x_i = 1$, $i = 1, \ldots, j$, $x_i = \sqrt{B_i(j)}$, $i = j+1, \ldots, n.$
3. If $B_n(n-1) < 1$, then $x_i = 1$, $i = 1, \ldots, n.$

In the paper [39], the problem of Saadany and Jaber (2008) [25] was solved by using this approach. The theorem can be used for the solution of more complicated models, with more activitites and more stock points. The general assumptions are formulated in Section 2. The general scheme is represented in Fig. 5. In the next section, the application of the GPRM will be demonstrated for the model [16].

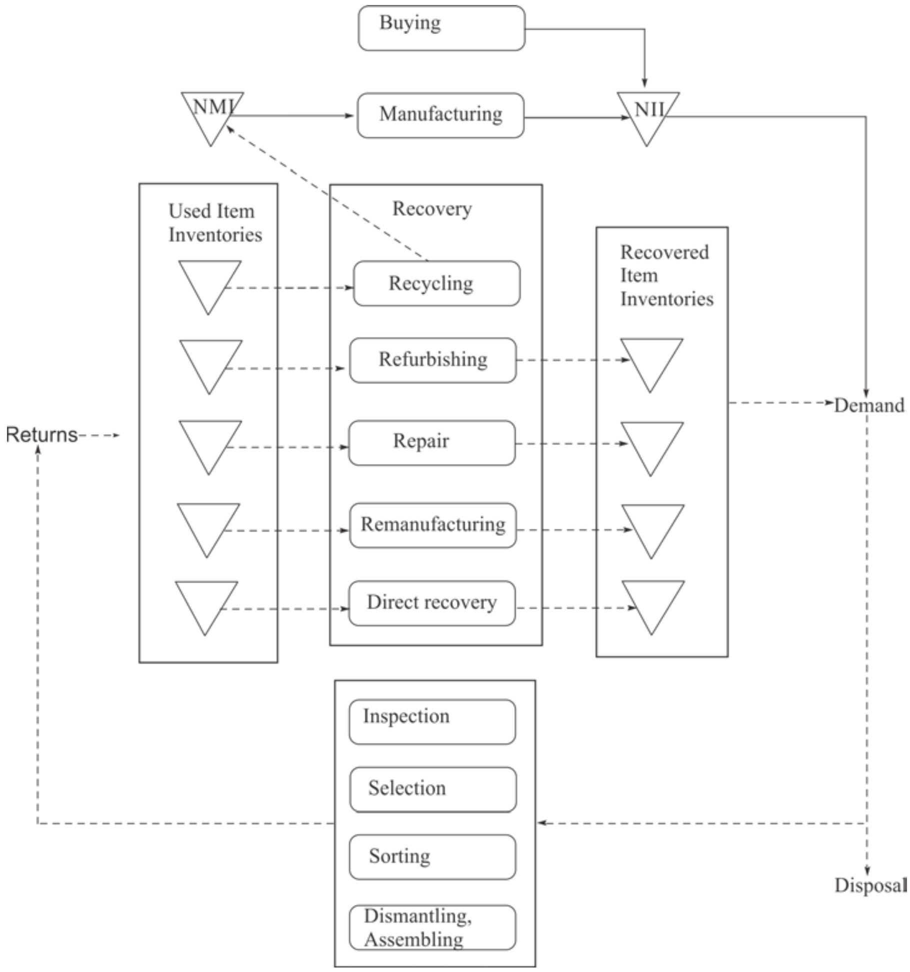

Fig. 5. Dynamics of inventories.

5 Application

In this section, the above discussed methodology is applied to the model [16].

5.1 Example 1: Lot sizing for a recoverable product with inspection, sorting, and switching cost [16]

Consider now an inventory system where demand is satisfied by recovered and new purchased items. Used units of a product returned by (or collected from) customers are kept in recoverable inventory until the start of a combined process of inspection and recovery. Remanufactured items are assumed to be as good as new. However, some recovered items do not qualify for classification as "remanufactured" and are perceived by customers to be of secondary quality. These refurbished items are sold to a secondary market at a reduced price. At any time when the production or recovery process is started, the changeover costs (costs of machine start-up) are incurred. This model is the extension of the works [11] and [16].

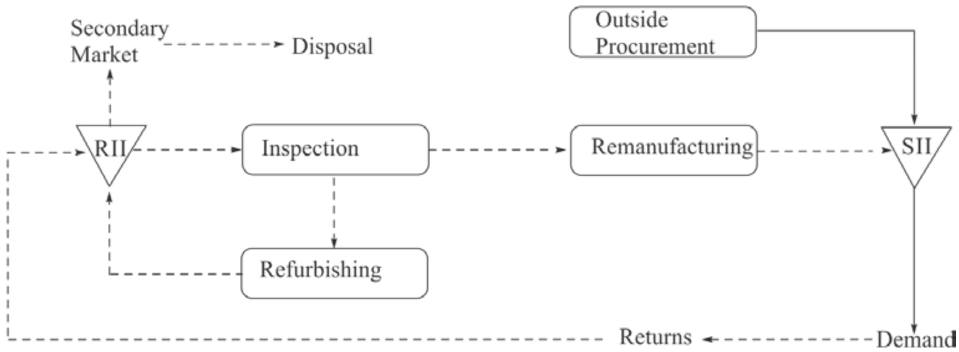

Figure 6: Framework of the inventory system.

Notation

Q^n	–	order quantity for new items,
Q^m	–	remanufacturing lot size
m	–	number of remanufacturing lots
n	–	number of lots of new items that are purchased
T	–	cycle time interval over which collection, inspection and sorting, refurbishing, remanufacturing, and ordering processes occur
d	–	constant demand rate
r	–	constant return rate
x	–	inspection and sorting rate (units per unit of time), $x > d > r$
P	–	changeover cost of starting up machines for remanufacturing at the beginning of the remanufacturing cycle
R	–	remanufacturing setup cost
S	–	ordering cost of a lot of new items
W	–	fixed inspection and sorting charge
c_1	–	unit remanufacturing cost
c_2	–	unit purchase cost of new items
c_3	–	unit refurbishing cost
c_4	–	unit inspection cost
h	–	holding cost of used and and refurbished items
H	–	holding cost of serviceable items
q	–	constant percentage of used items classified into the refurbishing category
s	–	unit selling price of serviceable (new and remanufactured) items

u – unit selling price of refurbished items, $u < s$

Suppose that $(1-q)x > d > r$.

$$\begin{aligned} nQ^n + mQ^m &= dT, \\ mQ^m &= (1-q)rT \end{aligned} \tag{21}$$

In the paper [16], two types of production and recovery policies were considered: an $(1,n)$-policy of a single inspection and sorting and a single recovery (remanufacturing and refurbishing) lot and n lots of new items manufactured; and an $(m,1)$-policy of m batches of recovery and of inspection and sorting and a single batch of new items for three cases of relations of initial parameters: $(1-q)x > D > r$, $D \geq (1-q)x > r$, and $D > r \geq (1-q)x$. The production scheduling is predetermined: ordering of a fixed number of lots of new items is followed by recovery of a fixed number of lots of used items per time cycle.

In our research, we consider the case of $(1-q)x > D > r$. The same production scheduling is used, in which the ordering of new items is followed by recovery of used items, and there are two types of control strategies: the push control strategy (in which all returned products are remanufactured as early as possible) and the pull control strategy (in which all returned products are remanufactured as late as is convenient).

The strategies are defined more formally in paper [41]. In the (s_m, Q_m, Q_n) PUSH-strategy, remanufacturing starts whenever the inventory of remanufacturables contains Q_m used products. In that case, all Q_m products enter the remanufacturing process to be remanufactured. Manufacturing takes place in batches of size Q_n and starts whenever the serviceable inventory position (serviceable inventory minus backlog plus all products in (re)manufacturing work in process) drops to the level s_m. The strategy is named the PUSH-strategy because the used products are pushed into the remanufacturing process as soon as possible independently from the actual demands and from the on-hand serviceable inventory. In this model we do not consider backlogging, lead times are equal to 0, the ordering of new items is instantaneous, and so we assume that $s_m = 0$. The dynamics of inventories under the push-strategy are represented in Fig. 6.

In the (s_m, Q_m, s_n, Q_n) PULL-strategy, remanufacturing starts whenever the serviceable inventory position is at or below s_m and sufficient remanufacturable inventory Q_m exists. Manufacturing starts whenever the serviceable inventory position drops to the level s_n. The manufacturing batch size is Q_n. It is assumed that $s_n \geq s_m$. The strategy is named the PULL-strategy because remanufacturable inventory is pulled into the remanufacturing process only when needed to fulfill customer demands for serviceables. In this paper it is assumed that $s_m = s_n = 0$. The dynamics of inventories under the pull-strategy are represented in Fig. 6.

Different strategies lead to different dynamics of inventory even if the scheduling is the same. There are two cases of inventory dynamics: I^{PUSH} (see Fig. 8) and I^{PULL} (see Fig. 7). Both inventory dynamics were already considered in paper [16], but I^{PULL} was considered under the policy $(1,n)$, and I^{PUSH} under the policy $(m,1)$. In this research, both inventories are considered under a more general policy (m,n).

Consider the case $(1-q)x > d > r$.

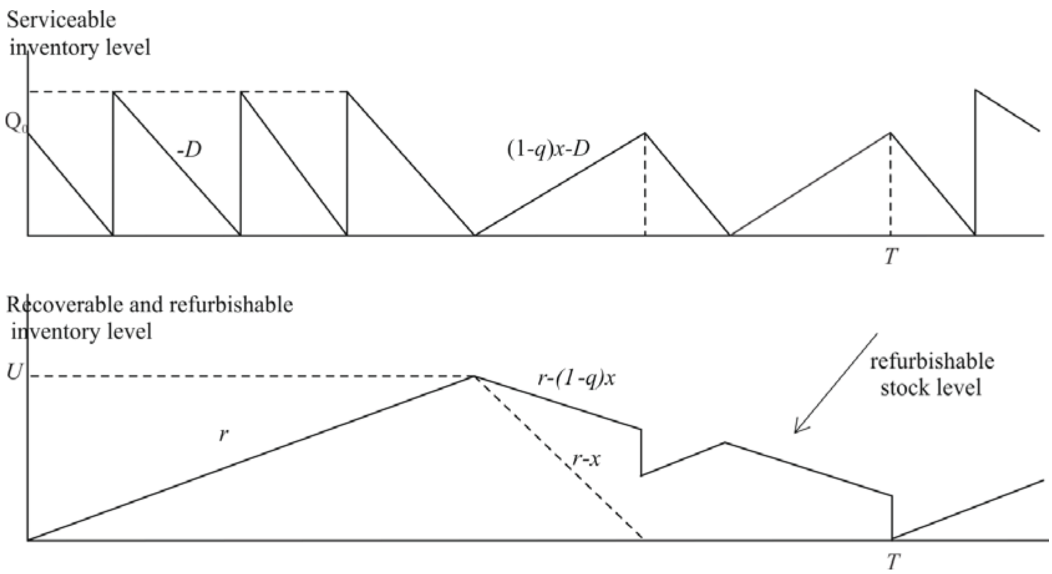

Figure 7: Serviceable and recoverable stock levels under the PULL strategy: $n = 3, m = 2$

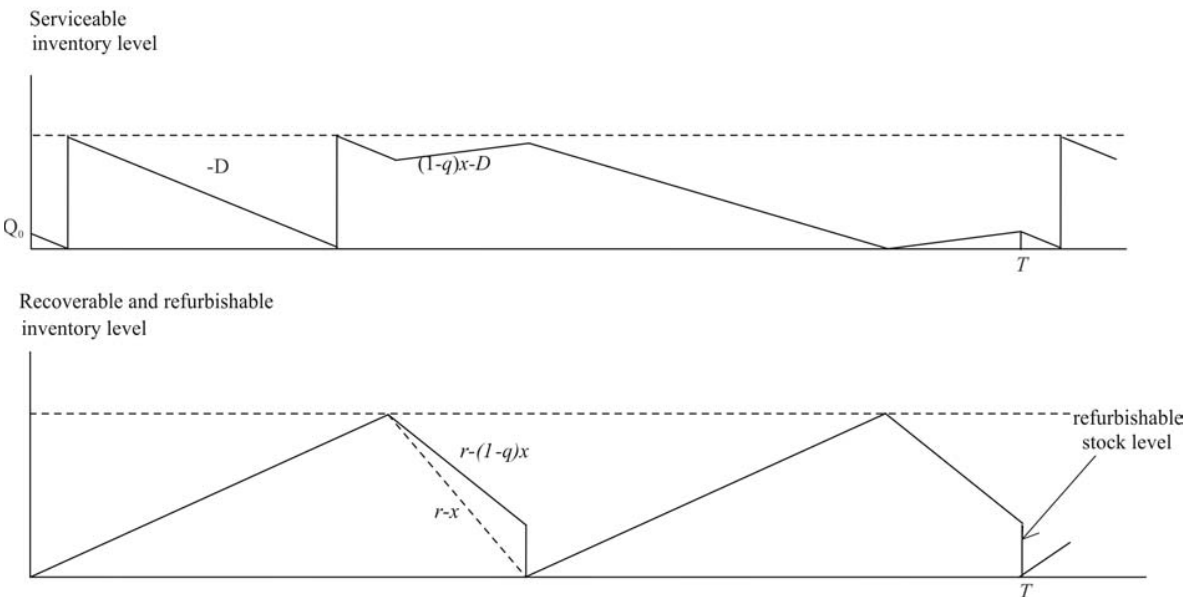

Figure 7: Recoverable and serviceable stock levels under the PUSH strategy: $n = 2, m = 2$

The first two cases were already considered in [16].

We denote TR and TC respectively as the total revenue and total cost per cycle. $TR(T)$ is the sum of revenues generated from selling new items nQ^n, remanufactured items $mQ^m = (1-q)rT$

, and refurbished qrT items and is given as

$$TR(T) = Tds + Trqu$$

$TC(T)$ is defined by (7), where $F \cdot T$ is the sum of purchasing cost $nQ^n c_2$, remanufacturing cost $T(1-q)rc_1$, refurbishing cost $Trqc_3$, inspection and sorting cost $Trqc_4$, holding cost per cycle $HC_I(T,m,n) = T \cdot H_I(m,n)$, remanufacturing fixed setup cost R, and inspection and sorting fixed cost W, and $G(m,n) = P + (W+R)m + nS$, and is given as:

$$F \cdot T = T(1-q)rc_1 + T(d-(1-q)r)c_2 + Trqc_3 + Trqc_4$$

$$ATC(T,m,n) = \frac{TC(T,m,n)}{T} = \frac{G(m,n)}{T} + T \cdot H_I(m,n) + F \cdot T$$

$$\begin{aligned}
&ATC(m,n) = F + \sqrt{H_I(m,n) \cdot G(m,n))} \\
&F = (1-q)rc_1 + (d-(1-q)r)c_2 + rqc_3 + rqc_4 \\
&G(m,n) = P + (W+R)m + nS
\end{aligned}$$

Under I^{PULL}, we have the following holding costs:

$$\begin{aligned}
&H_{I^{PULL}}(m,n) = h\left(\frac{r(d-(1-q)r)}{2d} - \frac{r^3((1-q)x-d)}{dx^2}\right) + \\
&+\frac{1}{m}[h\left(\frac{((1-q)x-d)r^3}{dx^2} + \frac{(1-q)(x-d)r^2}{2dx}\right) + \\
&+H\frac{(1-q)r^2((1-q)x-d)}{2dx}] + \frac{1}{n}H\frac{(d-(1-q)r)^2}{2d}
\end{aligned}$$

Coefficients $a_i, i = 1...3$, are as follows:

$$\begin{aligned}
a_1^{PULL} &= h\left(\frac{r(d-(1-q)r)}{2d} - \frac{r^3((1-q)x-d)}{dx^2}\right) \\
a_2^{PULL} &= [h\left(\frac{((1-q)x-d)r^3}{dx^2} + \frac{(1-q)(x-d)r^2}{2dx}\right) + H\frac{(1-q)r^2((1-q)x-d)}{2dx}] \\
a_3^{PULL} &= H\frac{(d-(1-q)r)^2}{2d}
\end{aligned}$$

Under I^{PUSH} we have the following holding costs:

$$H_{I_{PUSH}}(m,n) = H\frac{(1-q)r(d-(1-q)r)}{2d} + \frac{1}{m}[h\frac{r(x-(1-q)r)}{2x} +$$
$$+H\left(\frac{(1-q)r((1-q)r-d)}{2d} + \frac{(1-q)r^2((1-q)x-d)}{2xd}\right)] + \frac{1}{n}H\frac{(d-(1-q)r)^2}{2d}$$

The coefficients are given by

$$a_1^{PUSH} = H\frac{(1-q)r(d-(1-q)r)}{2d}$$
$$a_2^{PUSH} = [h\frac{r(x-(1-q)r)}{2x} + H\left(\frac{(1-q)r((1-q)r-d)}{2d} + \frac{(1-q)r^2((1-q)x-d)}{2xd}\right)]$$
$$a_3^{PUSH} = H\frac{(d-(1-q)r)^2}{2d}$$

Substituting the coefficients under the condition of the theorem, we can find optimal lot sizes for any set of initial parameters.

In paper [16], $P(1,n)$ under the PULL strategy and $P(m,1)$ under the PUSH strategy were considered. In this section, we consider more general cases of policies, taking into account switching costs. The question is which strategy is relevant, PULL or PUSH. It can be found that

$$\begin{aligned} & a_1^{PUSH} - a_1^{PULL} = a_2^{PULL} - a_2^{PUSH} = \\ & = (H(1-q)-h)\frac{r(d-(1-q)r)}{2d} + h\frac{r^3((1-q)x-d)}{dx^2} \end{aligned} \tag{31}$$

It follows from (31) and $a_3^{PUSH} = a_3^{PULL}$ that policy $P(1,n)$ under PUSH and PULL strategies leads to the same values of holding costs: $H_{I_{PUSH}}(1,n) = H_{I_{PULL}}(1,n)$. PUSH and PULL strategies lead to different results if $m > 1$. The PULL strategy will lead to lower costs ceteris paribus if $H(1-q) > h$.

To answer the question, we carry out a numerical analysis.

5.1.1 Numerical analysis

	h	H	x	d	r	q	P	W	R	S
Max.	20	20	20000	999	5000	0.5	100	100	100	100
Min.	1	1	5000	2000	1	0.01	1	1	1	1

Table 2: Example 1. Input parameters

The input parameters for the numerical analysis are listed in the table. Each of the model parameters has been set to vary in a range and is listed in the table.

Strategy	$P(n,m)$	$P(1,m)$	$P(n,1)$	$P(1,1)$	Total
PUSH	1255	722	17	0	1994
PULL	565	346	55	0	966
Indifferently	0	0	5461	1393	6854

Total	1820	1068	5533	1393	9814

Table 3: Example 1. Results, $P > 0$.

The sets of parameters $(h, H, x, D, r, q, P, W, R, S)$ for 10003 examples were randomly generated. When generating the d, x, r, and q values, the constraint $(1-q)x > D > r$ was respected. We compute and compare results for two policy for each data set. The results are listed in Table 2. In 189 cases, there are no solutions. In the other 9814cases, the policy $P(1,1)$ can be optimal if both policies have the same result. The PUSH strategy leads to the best result in 2165 cases, the PULL strategy in 966 cases; in the remaining 6872 cases, it makes no difference which strategy is used.

The results confirmed that $P(1,n)$ is optimal for 5533 examples (56.4%), $P(1,1)$ for 1393 (14.2%), $P(m,1)$ for 1068 (10.9%), and $P(m,n)$ for 1820 (18.5%).

Consider the same parameters, but with changeover cost equal to 0: $P = 0$. The results are listed in Table 3.

Strategy	$P(n,m)$	$P(1,m)$	$P(n,1)$	$P(1,1)$	Total
PUSH	0	628	0	0	628
PULL	0	332	0	0	332
Indifferently	0	0	5576	3278	8854
Total	0	960	5576	3278	9814

Table 4: Example 1. Results, $P = 0$.

6 Conclusion

Our paper considers a general class of deterministic inventory models with constant demand and return that generalizes the approach of Saadany and Jaber and contains other published types of models. Three types of costs are considered: first, the EOQ-unrelated cost, which is independent of the numbers of lots and lot sizes (the production cost), second, the EOQ-related cost that depends on the dynamics of the inventory, the lot sizes, and the numbers of lots (the holding cost), and third, the EOQ-related cost that depends on the numbers of lots and production scheduling (the switching cost).

The paper also proves the theorem that help to find the optimal policy under a predefined production schedule and control strategy, that is, the lot sizes and the optimal numbers of lots.

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