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Abstract. An economic lot size problem is studied in which a single vendor supplies a single purchaser with a homogeneous product and takes a certain fraction of the used items back for remanufacturing, in exchange for a deposit transferred to the purchaser. For the given demand, productivity, fixed ordering and setup costs, amount of the deposit, unit disposal, production and remanufacturing costs, and unit holding costs at the vendor and the purchaser, the cost-minimal order/lot sizes and remanufacturing rates are determined for the purchaser, the vendor, the whole system assuming partners' cooperation, and for a bargaining scheme in which the vendor offers an amount of the deposit and a remanufacturing rate, and the purchaser responds by setting an order size.

Keywords: Joint economic lot size, Reverse logistics, Closed loop supply chain, Collection, Remanufacturing, EOQ

1. Introduction

In the present paper, an economic lot size problem is studied in which a single vendor supplies a single purchaser with a homogeneous product and takes a certain fraction of the used items back for remanufacturing, in exchange for a deposit transferred to the purchaser. For the given demand, productivity, remanufacturing rate, fixed ordering and setup costs, amount of the deposit, unit disposal, production and remanufacturing costs, and unit holding costs at the purchaser and the vendor, the cost-minimal order/lot sizes and remanufacturing rates are determined for the purchaser, the vendor, the whole system assuming partners' cooperation, and for a bargaining scheme in which the vendor offers the amount of the deposit and the remanufacturing rate, and the purchaser reacts by setting the order size.

The problem of coordinating a supply chain with a single buyer demanding the product at a constant rate and a single supplier operating in a lot-for-lot fashion has been addressed by Monahan (1984) who considered quantity discounts as a coordination means. Subsequently Banerjee (1986a,b) has put forward a more general model that accounts for the inventory holding costs at the vendor, and coined the term *joint economic lot size*. Their work has triggered a broad research interest in this and related supply chain coordination problems. In particular, omission of the lot-for-lot assumption identifies a major research direction which addressed different forms of production and shipment policy (see, among others, Lee and Rosenblatt, 1986; Goyal, 1988; Hill, 1997, 1999; Eben-Chaime, 2004; Hill and Omar, 2006; Zhou and Wang, 2007; Hoque, 2009; see also Goyal, 1976, for an earlier model); we refer the reader to Sucky (2005) and Leng and Parlar (2005) for excellent reviews, as well. The work by Affisco et al. (2002) and Liu and Çetinkaya (2007) takes imperfect product quality into account and accommodates the opportunity of investing in quality improvement and setup cost reduction at the supplier. Kohli and Park (1989) have addressed the question of benefit sharing between the supply chain members from a game-theoretical standpoint. Sucky (2006) has further analysed the Banerjee's model in a setting with a privately informed buyer; similarly, Voigt and Inderfurth (2011) study a problem with information asymmetry and opportunity of investing in setup cost reduction. An approach to implement the joint optimization by the supply chain members without explicit disclosure of one's own sensitive data has been recently addressed by Pibernik et al. (2011).

At the same time, closed-loop supply chain problems are receiving an increasing attention in the literature. In the early work by Richter (1994, 1997) and Richter and Dobos (1999), an EOQ-like system with manufacturing, remanufacturing and disposal has been analyzed and the structure of optimal policies and cost functions established. More recently, Chung et al. (2008), Gou et al. (2008), Mitra (2009) and Yuan and Gao (2010) address the inventory costs minimization in a closed-loop supply chain context and offer an optimal manufacturing/remanufacturing strategy throughout the planning horizon. Bonney and Jaber (2011) extend the classical reverse logistics inventory models to environmentally related manufacturing factors in order to involve further metrics in the cost functions of inventory management — other than just the costs of operating the system. Grimes-Casey et al. (2007), Lee et al. (2010) and Subramoniam et al. (2010) investigate the practical application of reverse logistics models. In this context the models of Grimes-Casey et al. (2007) and Lee et al. (2010) analyze the lifecycle aspects of the reuse systems. Ahn (2009) and Grimes-Casey et al. (2007) suggest game-theoretical approaches for determining the best closed-loop supply chain strategies.

In the present paper we study a family of problems that represent, on the one hand, an extension of the basic model by Banerjee (1986a,b) to the case of reverse logistics flows — which have not been addressed in his work. On the other hand, they generalize the manufacturing and remanufacturing models of Richter (1994, 1997) and Richter and Dobos (1999) by accommodating a purchaser into the analysis (see Fig. 1).

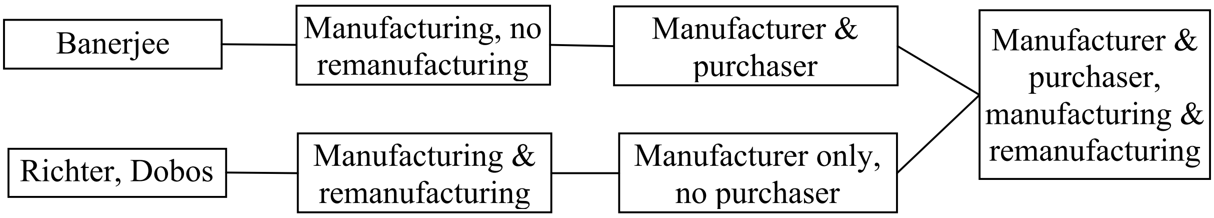

Fig. 1: Extending the models by Banerjee and Richter, Dobos

Furthermore, we investigate whether the reuse process helps to coordinate the vendor–purchaser supply chain. The paper is organized as follows. In the second section we present the models, classify them and construct the cost functions. In section 3 we determine the optimal lot sizes and minimum cost functions for the purchaser, the vendor and the total system for the case that the manufacturing process starts the production cycle. In the next section an auxiliary function is introduced. The explicitly found minimum point of this function helps to determine the optimal remanufacturing rates for the partners of the supply chain. In section 5 the same problems will be solved for the case of starting the production process with remanufacturing. In the sixth section, a bargaining scheme is considered in which the vendor offers a deposit amount and a remanufacturing rate, and the purchaser reacts by setting the order size. The final section concludes with a summary.

2. The model

2.1. General description

The model deals with the setting in which a single vendor (he) ships a homogeneous product to a single purchaser (her) according to her deterministic constant demand of D units per time unit. The vendor and the purchaser will have to agree on the regular shipment size q which has also to be the production lot size of the vendor. This size can be determined by the vendor, by the purchaser or jointly by both of them. The vendor manufactures new items and can also produce as-good-as-new items by remanufacturing used ones. Both kinds of items — which are shortly called *serviceables* — serve the demand of the purchaser. The end-of-use items (*nonserviceables*) can be collected by the purchaser and taken back to the vendor for remanufacturing. By assumption, the same vehicle that delivers the serviceables to the purchaser, also picks up the nonserviceables accumulated by her and returns them back to the vendor when backhauling (see Fig. 2). The parameters and decision variables employed in the model are presented in Fig. 3.

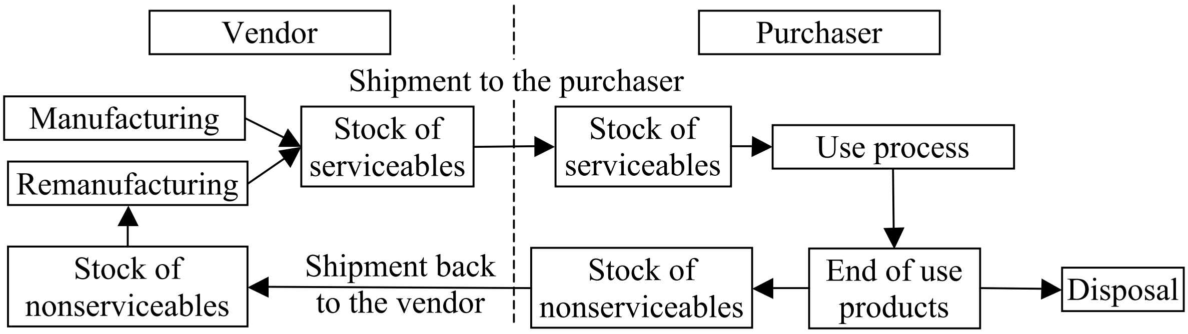

Fig. 2: Operations, transports and inventories in the closed loop chain

At both the vendor and the purchaser, inventories of serviceables and nonserviceables are held due to the rhythmic delivery of q units and the take back activities. The manufacturing and remanufacturing productivities P_M, P_R at the vendor's site, measured in units per time unit, are assumed to exceed the demand D, i. e., $\min(P_M, P_R) > D$. The vendor and the purchaser incur fixed costs s_v and s_p per order, respectively, and inventory holding costs $h_v > u_v$ and $h_p > u_p$ per unit of serviceables and nonserviceables per time unit, respsectively. The unit manufacturing and remanufacturing costs are respectively denoted by c_M, c_R . The purchaser is assumed to collect $\beta \cdot q$ units for the subsequent remanufacturing at the vendor's site, while the other part $(1-\beta)\cdot q$ is disposed at the cost c per unit. She receives d dollars per unit shipped back as a deposit returned by the vendor. The length of a cycle, which is the time between two shipments, is denoted by $T = q/D$. Parameter β is called *collection rate* for the purchaser and *remanufacturing rate* for the vendor.

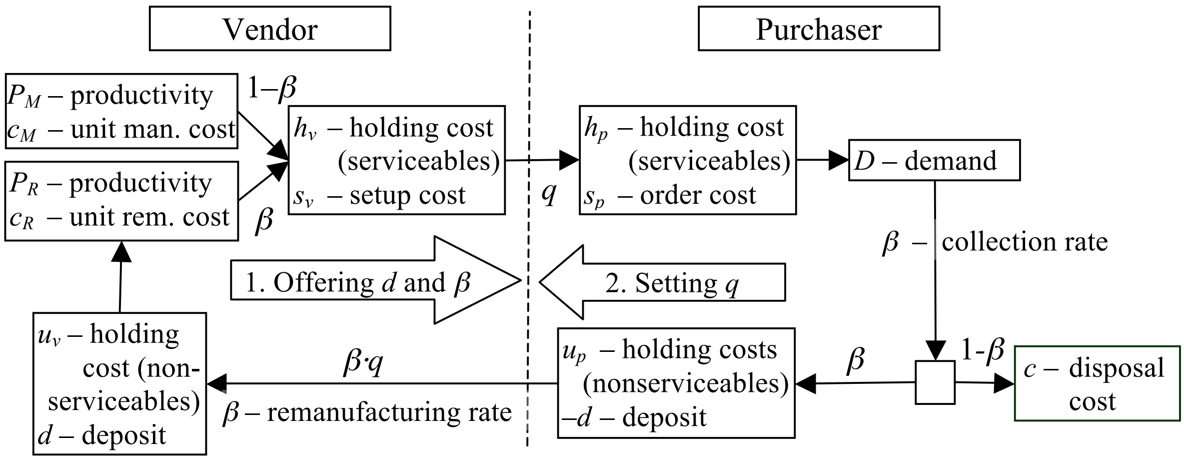

Fig. 3: Parameters and decision variables of the model

The following complex of problems will be subsequently studied (Fig. 4). The problems of the purchaser consist in determining her optimal order size and the corresponding minimum cost function for a given collection rate (problem PL) as well as determining the optimal collection rate which minimizes the minimum cost function (problem PC). The corresponding problems studied for the vendor have to be distinguished with respect to two options. If the vendor first manufactures new products and then remanufactures used ones (option MbR), his costs will be different compared to the costs due to the alternative sequence "Remanufacturing before manufacturing" (RbM). Like in the purchaser's case, the optimal lot size and the optimal collection rate will be determined, yet separately for the first option (problems VL-MbR and VC-MbR) and the second option (problems VL-RbM and VC-RbM). In the case of studying the vendor–purchaser system as a whole, the total of vendor's setup costs, his inventory holding costs for serviceables and nonserviceables, the purchaser's setup costs, her inventory holding costs for serviceables and nonserviceables, and her disposal costs, has to be minimized. Then the order of executing the manufacturing and remanufacturing processes is also significant, and therefore the respective problems SL-MbR, SC-MbR, SL-RbM and SC-RbM need to be considered. Finally the problems of determining the deposit and the collection (remanufacturing) rate which meet the purchaser's optimal order size VPC-MbR and VPC-RbM have to be studied.

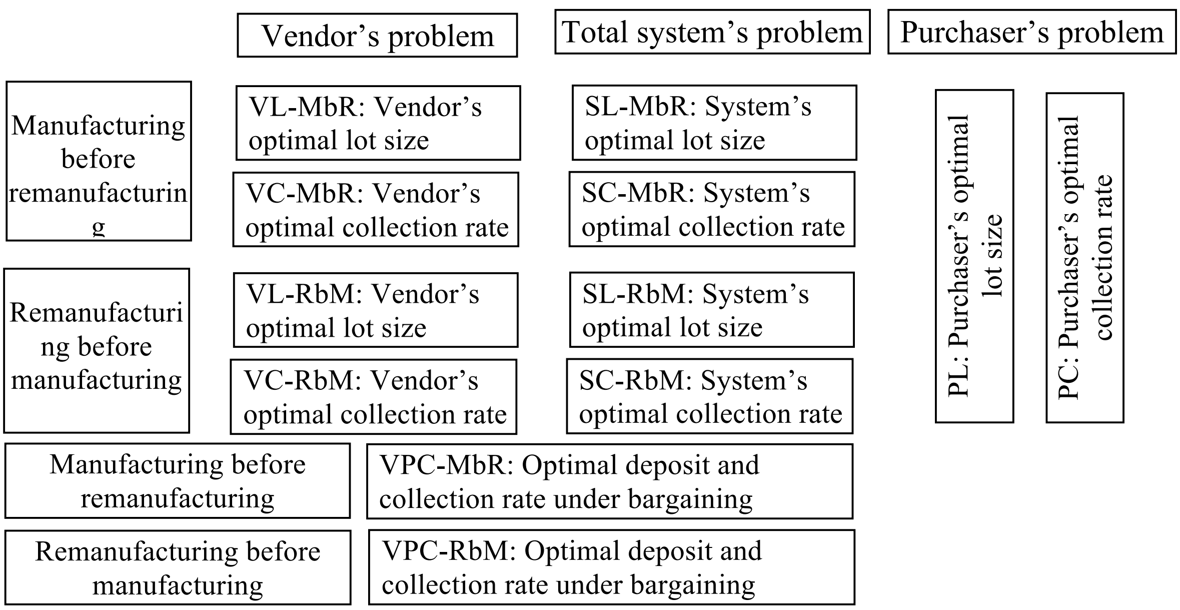

Fig. 4: Complex of problems studied in the paper

2.2. Purchaser's cost function

Before the models can be formulated analytically some further assumptions have to be made. The purchaser is supposed to sort out the items which are suitable for remanufacturing continuously over time and at a constant rate. Hence the average inventory level of her serviceables per order cycle is $I_p^s(q) = q/2$ and the inventory level of nonserviceables is $I_p^n(q,\beta) = \beta \cdot q/2$. Then the purchaser's total costs per time unit are expressed by

$$TC_p(q,\beta) = C_p(q,\beta) + R_p(\beta) \tag{1}$$

where

$$C_p(q,\beta) = s_p \cdot \frac{D}{q} + \frac{q}{2} \cdot (h_p + u_p \cdot \beta) \qquad \text{and} \qquad R_p(\beta) = (c - (c+d)\beta)D \tag{2}$$

represent the cost components which respectively do and do not depend on the order size.

2.3. Vendor's cost function for MbR

The inventory levels of serviceables and nonserviceables at the vendor depend obviously on the order in which manufacturing and remanufacturing are executed. Consider first the MbR option which assumes that in each order cycle, first $(1-\beta)q$ items are manufactured and then βq items are remanufactured. The RbM option will be studied subsequently in Section 5.

Figures 5 and 6 illustrate the evolution of the inventory level of serviceables and nonserviceables at the vendor during an order cycle, where t_2 and t_3 represent the duration of manufacturing and remanufacturing operations, respectively, and $t_1 = T - t_2 - t_3$ is the slack time. Note that $t_2 + t_3 < T$ holds due to the assumption $P_M, P_R > D$ made in Section 2.1.

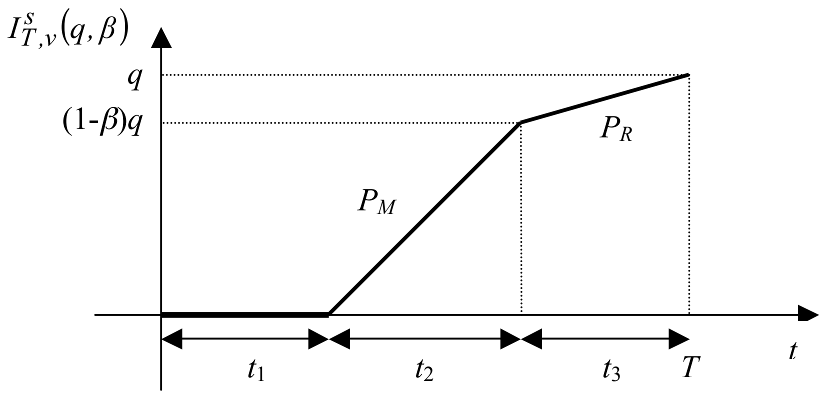

Fig. 5: The serviceables' inventory level at the vendor during an order cycle

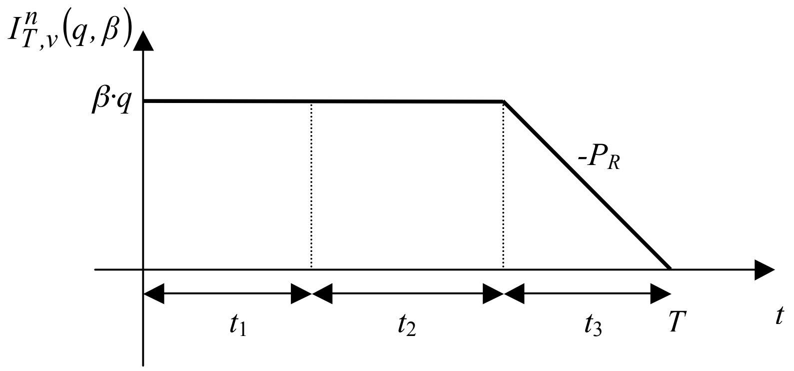

Fig. 6: The nonserviceables' inventory level at the vendor during an order cycle

Let $H_j(q,\beta)$, $j = s,n$, denote the holding costs per time unit for the vendor's serviceables and nonserviceables, respectively. It is straightforward to obtain that

$$H_s(q,\beta) = \frac{q}{2} h_v \cdot \left[(\beta - 2) \cdot \beta \cdot \left(\frac{D}{P_M} - \frac{D}{P_R} \right) + \frac{D}{P_M} \right],$$

$$H_n(q,\beta) = u_v \cdot \frac{q}{2} \cdot \left(2 \cdot \beta - \beta^2 \frac{D}{P_R} \right)$$

what in total gives the following expression of the vendor's total holding costs per time unit:

$$H_v(q,\beta) = \frac{q}{2} \cdot \left\{ h_v \cdot \left[(\beta - 2) \cdot \beta \cdot \left(\frac{D}{P_M} - \frac{D}{P_R} \right) + \frac{D}{P_M} \right] + u_v \cdot \left(2 \cdot \beta - \beta^2 \frac{D}{P_R} \right) \right\}$$

By rearranging the terms within the curly braces in the above cost expression, the following three terms can be identified: one that does not depend on the collection rate β, another one being linear in β, and the third one being quadratic in β, multiplied by the factors which we define respectively as

$$V = \frac{h_v \cdot D}{P_M}, \qquad \Omega_M = h_v \left(\frac{D}{P_M} - \frac{D}{P_R} \right) - u_v \quad \text{and} \quad \Delta_M = h_v \left(\frac{D}{P_M} - \frac{D}{P_R} \right) - u_v \frac{D}{P_R} \tag{3}$$

so that

$$H_v(q,\beta) = \frac{q}{2} \cdot \left\{ \Delta_M \beta^2 - 2\Omega_M \beta + V \right\}. \tag{4}$$

Note that $\Omega_M = -u_v$ and $\Delta_M = -u_v \dfrac{D}{P_R}$ if $P_M = P_R$. Furthermore, the assumption $D < P_R$ made in Section 2.1 obviously implies that $\Omega_M < \Delta_M < V$. Also note that (4) is non-negative for all $q \geq 0$ and $0 \leq \beta \leq 1$ as $H_s(q,\beta)$ and $H_n(q,\beta)$ are both non-negative by construction.

The vendor's ordering and holding costs per time unit express consequently as

$$C_v(q,\beta) = s_v \frac{D}{q} + \frac{q}{2} \cdot \left\{V + \beta^2 \cdot \Delta_M - 2 \cdot \beta \cdot \Omega_M\right\}. \tag{5}$$

The vendor incurs also costs which do not depend on the lot size, amounting per time unit to

$$R_v(\beta) = \left(c_M + \left(d + c_R - c_M\right)\beta\right)D.$$

Accordingly, the vendor's total costs per time unit are expressed by

$$TC_v(q,\beta) = C_v(q,\beta) + R_v(\beta). \tag{6}$$

2.3. The total system costs

Now the costs of the vendor and the purchaser are considered jointly. The joint ordering and holding costs per time unit are then expressed by

$$C_T(q,\beta) = \left(s_v + s_p\right)\frac{D}{q} + \frac{q}{2} \cdot \left\{\beta^2 \cdot \Delta_M + \beta\left(u_p - 2\Omega_M\right) + V + h_p\right\}. \tag{7}$$

Let

$$W = V + h_p \qquad \text{and} \qquad U = u_p - 2\Omega_M . \tag{8}$$

Then (7) can be rewritten as

$$C_T(q,\beta) = \left(s_v + s_p\right)\frac{D}{q} + \frac{q}{2} \cdot \left\{\beta^2 \cdot \Delta_M + \beta U + W\right\}$$

and the system-wide total costs per time unit expressed by

$$TC_T(q,\beta) = C_T(q,\beta) + R_T(\beta) \tag{9}$$

where $R_T(\beta) = \left(c + c_M + \beta\left(c_R - c - c_M\right)\right)D$ is the cost component that does not depend on the lot size. Note that

$$R_T(\beta) = R_v(\beta)\Big|_{d=0} + R_p(\beta)\Big|_{d=0} . \tag{10}$$

3. Analysis of the optimal lot sizes and minimal cost functions

Firstly, the cost functions of the purchaser, the vendor and the whole system are analyzed to determine the optimal order and lot sizes, respectively.

The purchaser's PL problem is merely a modification of the classical EOQ model. The optimal order size for a given collection rate β is then expressed as

$$q_p^*(\beta) = \sqrt{\frac{2 \cdot s_p \cdot D}{h_p + \beta \cdot u_p}} \tag{11}$$

with the corresponding minimum total costs

$$TC_p^*(\beta) = \sqrt{2 \cdot D \cdot s_p \cdot \left(h_p + \beta \cdot u_p\right)} + R_p(\beta). \tag{12}$$

In the same way, the vendor's optimal lot size under MbR policy (i.e., the optimal solution to the VL-MbR problem) is the one that minimizes (6) and expresses accordingly as

$$q_v^*(\beta) = \sqrt{\frac{2 \cdot D \cdot s_v}{\beta^2 \cdot \Delta_M - 2 \cdot \beta \cdot \Omega_M + V}}\,. \tag{13}$$

His resulting minimum total costs are then represented by the function

$$TC_v^*(\beta) = \sqrt{2 \cdot D \cdot s_v \cdot \left(\beta^2 \cdot \Delta_M - 2 \cdot \beta \cdot \Omega_M + V\right)} + R_v(\beta). \tag{14}$$

It is straightforward to see that the sign of this function's 2nd derivative does not depend on β, therefore (14) is either convex or concave in β. Thus by analyzing its 1st derivative and the function values at the borders, we can easily determine whether its minimum is attained at $\beta = 0$, $\beta = 1$, or some point in between. In the latter case, the optimal collection rate will be called *non-trivial*. Section 4.2 below provides a detailed analysis of the respective VC-MbR problem.

Similarly, it follows from (7)–(9) that the system-wide optimal lot size (i.e., the solution to the SL-MbR problem) is given by

$$q^*(\beta) = \sqrt{\frac{2 \cdot D \cdot (s_v + s_p)}{W + \beta^2 \cdot \Delta_M + \beta \cdot U}} \tag{15}$$

and implies the following minimum total costs for the system:

$$TC^*(\beta) = \sqrt{2 \cdot D \cdot \left(s_v + s_P\right) \cdot \left(W + \beta^2 \cdot \Delta_M + \beta \cdot U\right)} + R_T(\beta). \tag{16}$$

It can be seen that in all of the above three cases, the classical EOQ formula is employed — with merely straightforward modifications that reflect the composition of the cost function coefficients in the purchaser's, the vendor's, and the system-wide problems, respectively.

4. The optimal collection and remanufacturing rates

Before we proceed with the analysis of the optimal collection and remanufacturing rates for the purchaser, the vendor and the whole system, respectively, it makes sense to utilize the fact that the respective minimum cost functions (12), (14), (16) have a similar form and to introduce the following "omnibus" function:

$$K(\beta) = G\sqrt{A + B\beta^2 - 2C\beta} + E \cdot \beta + F \tag{17}$$

for which we further assume that $A, G > 0$ and that

$$A + B\beta^2 - 2C\beta > 0 \tag{18}$$

holds on the entire domain $0 \leq \beta \leq 1$. It is straightforward to see that $K(\beta)$ is strictly convex if and only if

$$AB > C^2, \tag{19}$$

and otherwise concave. If strictly convex, the function has a minimum in the interior of its domain whenever $K'(0) < 0$ and $K'(1) > 0$ — what, in turn, holds if and only if the inequalities

$$\frac{C - B}{\sqrt{A + B - 2C}} < \frac{E}{G} < \frac{C}{\sqrt{A}} \tag{20}$$

hold. In all other cases the minimum is found at one of the boundary points $\beta = 0$, $\beta = 1$. This provides us with the following

Lemma 1. Assume that $AB > C^2$. If condition (20) is satisfied then $K(\beta)$ has a global minimum at

$$\beta^* = \frac{C}{B} - \frac{E}{B}\sqrt{\frac{AB - C^2}{BG^2 - E^2}} \tag{21}$$

and otherwise at

$\beta^* = 0$ whenever the right-hand inequality in (20) does not hold, and at

$\beta^* = 1$ whenever the left-hand one does not.

Proof: As discussed above, the assumption of the lemma implies that $K(\beta)$ is strictly convex. Let (20) hold. Then the function is decreasing at $\beta = 0$ due to $K'(0) < 0$ and increasing at $\beta = 1$ due to $K'(1) > 0$. By the strict convexity, the global minimum is attained at the unique interior point $\beta^* \in (0,1)$ with $K'(\beta^*) = 0$, which we accordingly determine by solving

$$K'(\beta) = G\frac{B\beta - C}{\sqrt{A + B\beta^2 - 2C\beta}} + E = 0 \tag{22}$$

what implies:

$$\frac{C^2-2BC\beta+B^2\beta^2}{A+B\beta^2-2C\beta}=\left(\frac{E}{G}\right)^2$$

and gives the following quadratic equation after rearranging the terms:

$$B(BG^2-E^2)\beta^2-2C(BG^2-E^2)\beta+C^2G^2-AE^2=0.$$

Note that $B>0$ holds due to the assumption of the lemma. Also note that $BG^2-E^2\neq 0$ since otherwise $C^2G^2=AE^2$ what contradicts (20). Hence the above equation can be rewritten as

$$\beta^2-2\frac{C}{B}\beta+\frac{C^2G^2-AE^2}{B(BG^2-E^2)}=0.$$

Its roots are given by

$$\beta_{1,2}=\frac{C}{B}\pm\sqrt{\frac{C^2}{B^2}-\frac{C^2G^2-AE^2}{B(BG^2-E^2)}}=\frac{C}{B}\pm\sqrt{\frac{ABE^2-C^2E^2}{B^2(BG^2-E^2)}}=\frac{C}{B}\pm\frac{|E|}{B}\sqrt{\frac{AB-C^2}{BG^2-E^2}}. \quad (23)$$

Substituting both roots for β in (22), one can see that $\beta^*=\beta_2$ if $E>0$, otherwise $\beta^*=\beta_1$. Thus the following applies:

$$\beta^*=\frac{C}{B}-\frac{E}{B}\sqrt{\frac{AB-C^2}{BG^2-E^2}}.$$

Assume now that the right-hand inequality in (20) does not hold — what implies $K'(0)\geq 0$. By the strict convexity, $K''>0$ and thus K' is growing at each point of the domain. Therefore $K'(\beta)>0$ at all $0<\beta\leq 1$, what proves $K(\beta)$ to grow on the right from $\beta=0$. Then obviously $\beta^*=0$ is its global minimum. By a similar argument we obtain $\beta^*=1$ when the left-hand inequality in (20) does not hold. Note that with $K''>0$, both inequalities in (20) cannot be violated simultaneously. $\square$

It follows from the above proof that real-valued roots (23) which solve (22) exist under the assumption of Lemma 1 if and only if $BG^2>E^2$, otherwise there is no such interior point at which $K'(\beta)=0$. This provides us with the following

Corollary 1. The condition of Lemma 1 can only be satisfied if $B>\frac{E^2}{G^2}$ holds. A sufficient (but not necessary) condition for that is $C>0, E\geq 0$, provided that the assumption and the condition of Lemma 1 hold.

Proof: Indeed, assume that $C>0$ and $E\geq 0$ holds. The condition of Lemma 1 requires $\frac{E}{G}<\frac{C}{\sqrt{A}}$, what is then equivalent to requiring $\frac{E^2}{G^2}<\frac{C^2}{A}$, given that $A,G>0$ holds for $K(\beta)$ by assumption. At the same time, the assumption of the Lemma implies $\frac{C^2}{A}<B$. This gives $\frac{E^2}{G^2}<B$, as required. □

The following lemma treats the case of concave $K(\beta)$.

Lemma 2. If $AB\leq C^2$ then $K(\beta)$ has a global minimum at $\beta^*=0$ whenever the inequality

$$\sqrt{A}-\sqrt{A+B-2C}\leq\frac{E}{G}$$

holds, and at $\beta^*=1$ whenever it holds in the opposite direction. The case of equality implies that both $\beta=0$ and $\beta=1$ deliver a global minimum to $K(\beta)$.

Proof: The condition $AB\leq C^2$ implies that the 2[nd] derivative K'' is non-negative and constant everywhere on the function's domain, hence $K(\beta)$ is either strictly concave or linear. Hence if a local optimum is found in the domain's interior, it can only be a global maximum; otherwise the function is monotonic on the entire domain and, in either case, it suffices to compare its values at the boundary points to figure out a global minimum. This immediately leads to the assertion of the lemma. □

We now proceed to the analysis of the respective optimal collection rates for the purchaser, the vendor and the whole system.

4.1. Purchaser's solution

It is straightforward to see that the purchaser's minimum cost function (12) represents $K(\beta)$ with $A=h_p$, $B=0$, $C=-u_p/2$, $E=-(c+d)D$, $F=cD$, $G=\sqrt{2Ds_p}$ and obviously complies with the assumptions made for $K(\beta)$ at the beginning of Section 4. It is also obvious to see that it satisfies the condition of Lemma 2, hence the purchaser is not interested in collecting any used items ($\beta^*=0$) if

$$(c+d)\sqrt{D}\quad<\quad\left(\sqrt{h_p+u_p}-\sqrt{h_p}\right)\cdot\sqrt{2s_p}$$

and will collect all of them ($\beta^*=1$) in the opposite case. He is indifferent between the two policies if the above inequality be equality.

4.2. Vendor's solution (VC-MbR)

The vendor's minimum cost function (14) represents $K(\beta)$ with $A = V$, $B = \Delta_M$, $C = \Omega_M$, $E = (d + c_R - c_M)D$, $F = c_M D$ and $G = \sqrt{2Ds_v}$. There holds accordingly the following

Lemma 3. Under conditions (18)–(20), the vendor's optimal remanufacturing rate is non-trivial and given by

$$\beta^* = \frac{\Omega_M}{\Delta_M} - \frac{E}{\Delta_M}\sqrt{\frac{\Delta_M \cdot V - \Omega_M^2}{\Delta_M \cdot 2Ds_v - E^2}}. \tag{24}$$

Corollary 2. Equation (24) can be used to determine the vendor's optimal remanufacturing rate whenever the following conditions simultaneously hold:

$$\Delta_M > \frac{\Omega_M^2}{V} \tag{25}$$

$$\Delta_M > \frac{D \cdot (d + c_R - c_M)^2}{2s_v} \tag{26}$$

$$\frac{u_v \cdot (D/P_R - 1)}{\sqrt{(h_v - u_v) \cdot D/P_R + 2u_v}} < \frac{(d + c_R - c_M) \cdot D}{\sqrt{2Ds_v}} < \frac{\Omega_M}{\sqrt{V}} \tag{27}$$

In particular, either of conditions (25) and (26) imply:

$$\frac{P_R}{P_M} > 1 + \frac{u_v}{h_v}. \tag{28}$$

Hence P_R must necessarily exceed P_M for β^* being a non-trivial remanufacturing rate. If any of conditions (25)–(27) is violated then the vendor's optimal remanufacturing rate happens to be either $\beta^* = 0$ or $\beta^* = 1$.

Proof: It can be easily seen that conditions (19) and (20) immediately translate to (25) and (27), whereas (18) translates to:

$$\Delta_M \beta^2 - 2\Omega_M \beta + V > 0.$$

It is straightforward to see that the quadratic function on the left has no real-valued roots whenever $\Omega_M^2 < \Delta_M V$ — what is, in turn, required by (25). Also by the virtue of (25), $\Delta_M > 0$ and the quadratic function has its branches directed upwards. This guarantees the required positivity of the function for all feasible values of β; thus, (25) implies (18) to hold. Further, condition (26) is an immediate consequence of Corollary 1. Finally, either of (25) and (26) imply $\Delta_M > 0$. This together with the definition of Δ_M in (3) leads to (28). $\square$

Example 1 (VC-MbR)**.** Let $D = 100$, $s_v = 1000$, $h_v = 100$, $u_v = 5$, $c_M = 35$, $c_R = 20$, $d = 17$, $P_M = 200$, $P_R = 250$ (see Fig. 7). Then $A = V = 50$, $B = \Delta_M = 8$, $C = \Omega_M = 5$, $E = 200$, $F = 3500$, $G \approx 447.21$. It is straightforward to see that conditions (25), (26), (27) respectively hold:

$$\Delta_M = 8 \quad > \quad \frac{\Omega_M^2}{V} = \frac{25}{50} = 0.5$$

$$\Delta_M = 8 \quad > \quad \frac{D\cdot (d + c_R - c_M)^2}{2s_v} = \frac{100\cdot (17+20-35)^2}{2000} = 0.2$$

$$\frac{u_v \cdot (D/P_R - 1)}{\sqrt{(h_v - u_v)\cdot D/P_R + 2u_v}} \approx -0.433 \quad < \quad \frac{(d + c_R - c_M)\cdot D}{\sqrt{2Ds_v}} \approx 0.447 \quad < \quad \frac{\Omega_M}{\sqrt{V}} \approx 0.707$$

Hence by Corollary 2, the vendor's optimal remanufacturing rate is non-trivial and found to be $\beta^* \approx 0.237$ with the associated vendor's total costs $TC_v^*(\beta^*) \approx 6648.35$, see Fig. 7. Fig. 8 tracks β^* and $TC_v^*(\beta^*)$ for a range of values of deposit d.

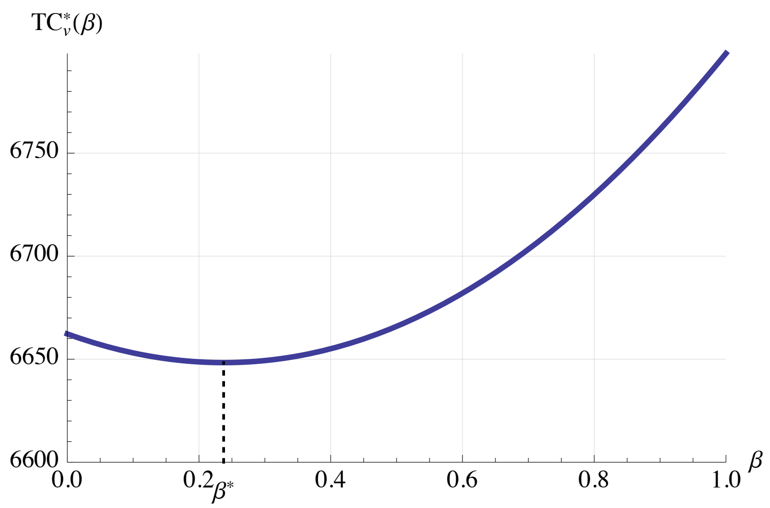

Fig. 7: The minimum cost curve $TC_v^*(\beta)$ and the non-trivial optimal remanufacturing rate of Example 1

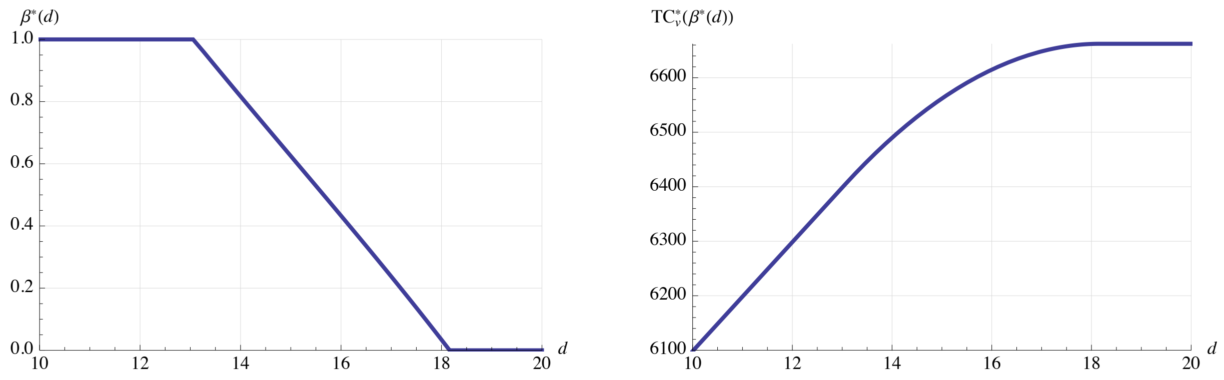

Fig. 8: The optimal collection rate (left) and the vendor's minimum total costs (right) as functions of the deposit in Example 1

4.3. The total system solution (SC-MbR)

The system-wide cost function (16) represents $K(\beta)$ with $A = W$, $B = \Delta_M$, $C = \frac{-U}{2} = \Omega_M - \frac{u_p}{2}$, $E = (c_R - c - c_M)D$, $F = (c_M + c)D$ and $G = \sqrt{2D(s_v + s_p)}$.

Lemma 4. Under conditions (18)–(20), the system-optimal collection and remanufacturing rate is non-trivial and given by

$$\beta_T^* = \frac{\Omega_M}{\Delta_M} - \frac{E}{\Delta_M}\sqrt{\frac{\Delta_M \cdot W - (\Omega_M - u_p/2)^2}{\Delta_M \cdot 2D(s_v + s_p) - E^2}} - \frac{u_p}{2\Delta_M}. \tag{29}$$

Corollary 3. Equation (29) can be used to determine the system-optimal collection and remanufacturing rate whenever the following conditions simultaneously hold:

$$\Delta_M > \frac{(\Omega_M - u_p/2)^2}{V + h_p} \tag{30}$$

$$\Delta_M > \frac{D \cdot (c_R - c - c_M)^2}{2(s_v + s_p)} \tag{31}$$

$$\frac{u_v \cdot (D/P_R - 1) - u_p/2}{\sqrt{(h_v - u_v) \cdot D/P_R + 2u_v + h_p + u_p}} < \frac{(c_R - c - c_M) \cdot D}{\sqrt{2D(s_v + s_p)}} < \frac{\Omega_M - u_p/2}{\sqrt{V + h_p}} \tag{32}$$

In particular, either of conditions (30) and (31) imply (28) and, hence, P_R must necessarily exceed P_M for β_T^* being a non-trivial collection and remanufacturing rate. If any of (30)–(32) is violated then the system-optimal collection and remanufacturing rate is either $\beta_T^* = 0$ or $\beta_T^* = 1$.

Proof: analogous to the proof of Corollary 2.

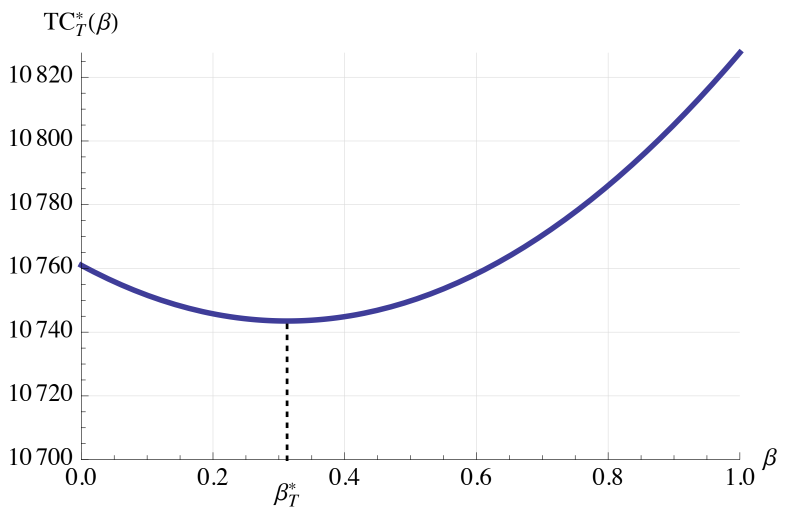

Fig. 9: The minimum cost curve $TC_T^*(\beta)$ and the non-trivial system-optimal collection rate of Example 2

Note that equations (24) and (29) coincide if $s_p = u_p = 0$ and $c_M + c - c_R = d + c_R - c_M$.

Example 2. Let the same data as in Example 1 hold with the following modification: $c_R = 45$, and further let $s_p = 400$, $h_p = 90$, $u_p = 5$, $c = 10$. Then the system-wide optimal collection and remanufacturing rate is $\beta_T^* \approx 0.3125$ with the associated total costs $TC_T^*(\beta_T^*) \approx 10743.5$, see Fig. 9.

5. The case of remanufacturing before manufacturing (RbM)

5.1. Vendor's inventory costs

Consider first the vendor's problem. Due to the assumption that remanufacturing precedes manufacturing, the inventory levels of serviceables and nonserviceables during an order cycle evolve as illustrated in Fig. 10 and 11, respectively. The vendor's total inventory holding costs per time unit express accordingly as follows:

$$H_s(q,\beta) + H_n(q,\beta) = \frac{q}{2}\cdot\left\{h_v\cdot\left[\beta^2\cdot\left(\frac{D}{P_R}-\frac{D}{P_M}\right)+\frac{D}{P_M}\right]+u_v\cdot\left[\beta^2\cdot\left(2\cdot\frac{D}{P_M}-\frac{D}{P_R}\right)+\beta\cdot\left(2-2\cdot\frac{D}{P_M}\right)\right]\right\} =$$

$$= \frac{q}{2}\cdot\left\{h_v\cdot\left[\beta^2\cdot\frac{D}{P_R}+(1-\beta^2)\cdot\frac{D}{P_M}\right]+u_v\cdot\beta\cdot\left[2-2\cdot(1-\beta)\cdot\frac{D}{P_M}-\beta\cdot\frac{D}{P_R}\right]\right\} \quad (33)$$

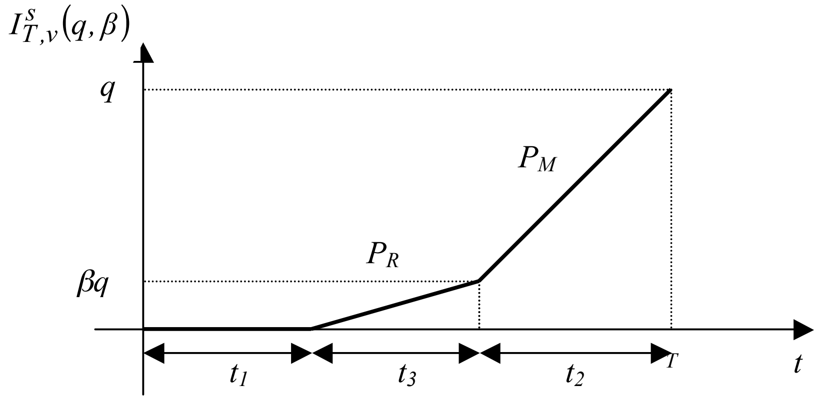

Fig. 10: Evolution of the serviceables' inventory level during an order cycle

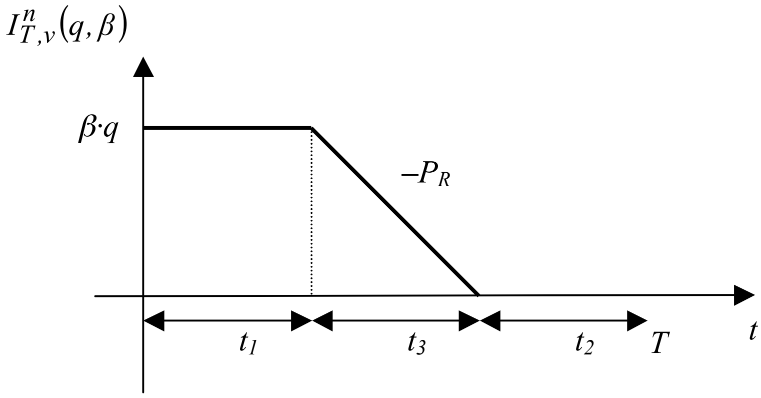

Fig. 11: Evolution of the nonserviceables' inventory level during an order cycle

By comparing vendor's holding cost expressions (3)–(4) and (33) in MbR and RbM settings, respectively, it is straightforward to obtain the following

Lemma 5. Let the same order size q and remanufacturing rate β apply in MbR and RbM settings. Then:

1. The vendor's total holding costs in the MbR setting are strictly below those in the RbM setting if and only if

$$\frac{P_R}{P_M} > 1 + \frac{u_v}{h_v - u_v} \tag{34}$$

2. They are equal if and only if (34) holds as equality or $\beta \in \{0, 1\}$.

Corollary 4. The MbR setting outperforms the RbM one if the remanufacturing productivity sufficiently exceeds the manufacturing one.

Corollary 5. Note that (34) represents a stronger condition than (28) which is necessary for a non-trivial optimal remanufacturing rate in the MbR setting, from the vendor's perspective.

5.2. Vendor's solution (VC-RbM)

As in Section 2, let us define

$$\Delta_R = (h_v - u_v)\left(\frac{D}{P_R} - \frac{D}{P_M}\right) + u_v \frac{D}{P_M} \qquad \text{and} \qquad \Omega_R = \left(1 - \frac{D}{P_M}\right) \cdot u_v . \tag{35}$$

Then the vendor's holding costs (33) can be rewritten as

$$H_v(q,\beta) = \frac{q}{2} \cdot \left\{V + \beta^2 \Delta_R + 2\beta\Omega_R\right\}, \tag{36}$$

his total costs accordingly as

$$TC_v(q,\beta) = \frac{s_v D}{q} + \frac{q}{2} \cdot \left\{V + \beta^2 \Delta_R + 2\beta\Omega_R\right\} + R_v(\beta) ,$$

and his minimum total costs for a given remanufacturing rate as

$$TC_v^*(\beta) = \sqrt{2 \cdot D \cdot s_v \cdot \left(V + \beta^2 \cdot \Delta_R + 2\beta\Omega_R\right)} + R_v(\beta), \tag{37}$$

The latter expression represents function $K(\beta)$ as defined in (17), with $A = V$, $B = \Delta_R$, $C = -\Omega_R$, $E = (d + c_R - c_M)D$, $F = c_M D$ and $G = \sqrt{2 \cdot D \cdot s_v}$.

Lemma 6. Under conditions (18)–(20), the vendor's optimal remanufacturing rate in the RbM setting is non-trivial and given by

$$\beta^*_{RbM} = -\frac{\Omega_R}{\Delta_R} - \frac{E}{\Delta_R}\sqrt{\frac{\Delta_R \cdot V - \Omega_R^2}{\Delta_R \cdot 2Ds_v - E^2}} . \tag{38}$$

Corollary 6. Equation (38) can be used to determine the vendor's optimal remanufacturing rate whenever the following conditions simultaneously hold:

$$\Delta_R > \frac{\Omega_R^2}{V} \tag{39}$$

$$\Delta_R > \frac{D \cdot (d + c_R - c_M)^2}{2s_v} \tag{40}$$

$$\frac{(h_v - u_v)\left(D/P_R - D/P_M\right) + u_v}{\sqrt{(h_v - u_v)D/P_R + 2u_v}} > \frac{(c_M - d - c_R)D}{\sqrt{2Ds_v}} > \frac{\Omega_R}{\sqrt{V}} \tag{41}$$

In particular, either of conditions (39) and (40) imply:

$$\frac{P_M}{P_R} > 1 - \frac{u_v}{h_v - u_v}. \tag{42}$$

Hence given a relatively low holding cost rate u_v of nonserviceables, the manufacturing productivity P_M must necessarily be above a sufficiently large fraction of the remanufacturing productivity P_R for β^* being a non-trivial remanufacturing rate. Furthermore, condition (41) implies:

$$c_M > c_R + d.$$

If any of conditions (39)–(41) is violated then the vendor's optimal remanufacturing rate happens to be either $\beta^* = 0$ or $\beta^* = 1$.

Proof: analogous to the proof of Corollary 2.

Example 3. Let $D = 2000$, $s_v = 500$, $h_v = 200$, $u_v = 120$, $c_M = 40$, $c_R = 20$, $d = 15$, $P_M = 3000$, and $P_R = 2500$. Then

$$\frac{P_R}{P_M} = 0.83 \quad < \quad 1 + \frac{u_v}{h_v - u_v} \approx 2.5$$

and by Lemma 5, the RbM setting outperforms its MbR counterpart for every fixed order size q and remanufacturing rate $\beta \in (0,1)$. Further, $V \approx 133.33$, $\Delta_R \approx 90.67$, $\Omega_R = 40$, $E = -10000$, and it is straightforward to see that conditions of Corollary 6 are satisfied, what results in $\beta^*_{RbM} \approx 0.81$ and the vendor's minimum total costs $TC^*_v(\beta^*_{RbM}) \approx 94598.88$. Fig. 12 displays the vendor's total costs for all $\beta \in [0,1]$ in both RbM and MbR settings, respectively assuming the optimal choice of the order size. As one can see, conditions of Corollary 2 are not satisfied for the MbR setting in the given example, leading to the optimal remanufacturing rate $\beta^*_{MbR} = 1$ in that setting.

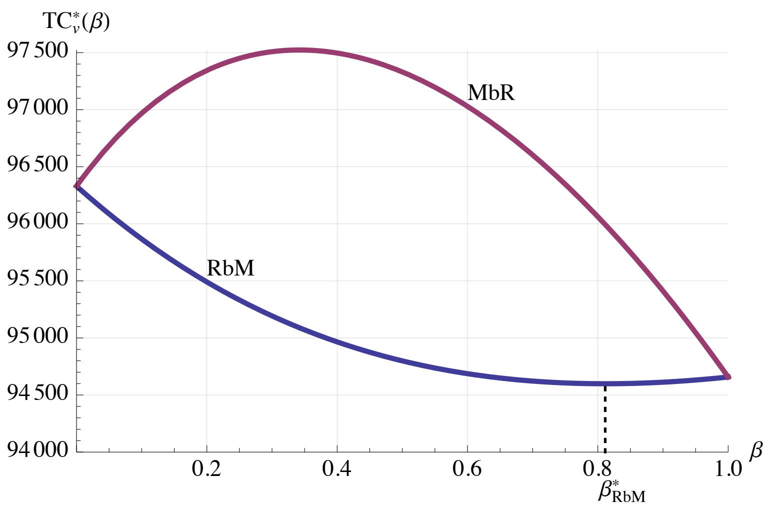

Fig. 12: Vendor's cost functions in the RbM and MbR settings of Example 3

5.3. The total system solution (SC-RbM)

It is straightforward to see that the system-wide costs in the RbM setting, assuming the optimal choice of the order size, are expressed as

$$TC_T^*(\beta) = \sqrt{2 \cdot D \cdot \left(s_v + s_p\right) \cdot \left(W + \beta^2 \cdot \Delta_R + \beta\left(2\Omega_R + u_p\right)\right)} + R_T(\beta) \tag{43}$$

and represent $K(\beta)$ with $A = W$, $B = \Delta_R$, $C = U = -\Omega_R - u_p/2$, $E = \left(c_R - c - c_M\right)D$, $F = \left(c_M + c\right)D$ and $G = \sqrt{2 \cdot D \cdot \left(s_v + s_p\right)}$.

Lemma 7. Under conditions (18)–(20), the system-optimal collection and remanufacturing rate in the RbM setting is non-trivial and given by

$$\beta^*_{T,RbM} = -\frac{\Omega_R}{\Delta_R} - \frac{E}{\Delta_R}\sqrt{\frac{\Delta_R \cdot W - (\Omega_R + u_p/2)^2}{\Delta_R \cdot 2D(s_v + s_p) - E^2}} - \frac{u_p}{2\Delta_R}. \tag{44}$$

Corollary 7. Equation (44) can be used to determine the system-optimal collection and remanufacturing rate whenever the following conditions simultaneously hold:

$$\Delta_R > \frac{(\Omega_R + u_p/2)^2}{V + h_p} \tag{45}$$

$$\Delta_R > \frac{D \cdot (c_R - c - c_M)^2}{2(s_v + s_p)} \tag{46}$$

$$-\frac{u_v + u_p/2 + (h_v - u_v)(D/P_R - D/P_M)}{\sqrt{(h_v - u_v) \cdot D/P_R + 2u_v + h_p + u_p}} > \frac{(c + c_M - c_R) \cdot D}{\sqrt{2D(s_v + s_p)}} > \frac{\Omega_M + u_p/2}{\sqrt{V + h_p}} \tag{47}$$

In particular, either of conditions (45) and (46) imply:

$$\frac{P_M}{P_R} > 1 - \frac{u_v}{h_v - u_v} \tag{48}$$

what always holds if $u_v \geq h_v/2$, and is otherwise equivalent to

$$\frac{P_R}{P_M} < 1 + \frac{u_v}{h_v - u_v} \tag{49}$$

what is at the same time necessary and sufficient for RbM to outperform MbR for every fixed order size q and remanufacturing rate $\beta \in (0,1)$. Furthermore, condition (47) implies:

$$c_R < c + c_M.$$

If any of (45)–(47) is violated then the system-optimal collection and remanufacturing rate $\beta^*_{T,RbM}$ is either 0 or 1.

Proof: analogous to the proof of Corollary 2 and by using Lemma 5.

Example 4. Let $D = 1000$, $s_v = 900$, $s_p = 400$, $h_v = 200$, $h_p = 220$, $u_v = 30$, $u_p = 40$, $c_M = 20$, $c_R = 33$, $c = 15$, $P_M = 2500$, $P_R = 1200$. Then the system-optimal collection and remanufacturing rate is $\beta^*_{T,RbM} \approx 0.2$ and the associated total costs $TC^*_{T,RbM}(\beta^*_{T,RbM}) \approx 62782.4$. Fig. 13 shows the minimum system-wide costs as function of β in both MbR and RbM settings.

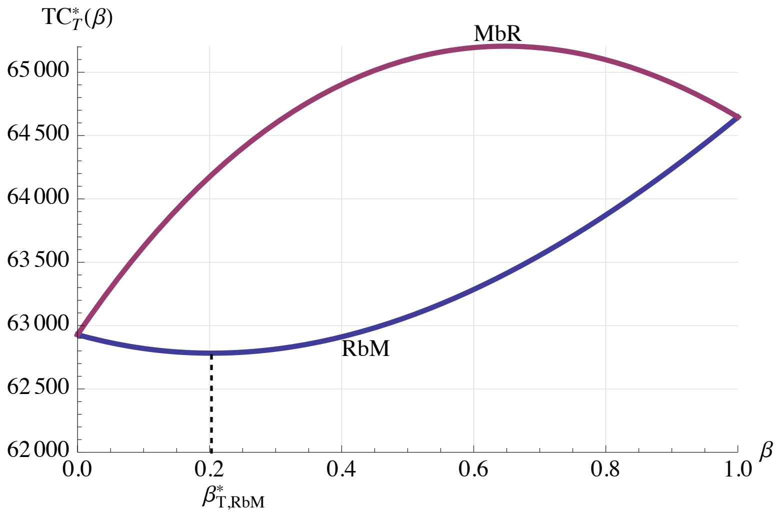

Fig. 13: System-wide cost function $TC^*_T(\beta)$ in the MbR and RbM settings of Example 4

6. Bargaining on the deposit and the collection rate: the VPC-MbR problem

In this section we study a bargaining solution for the MbR setting, assuming that the purchaser and the vendor are not vertically integrated and pursue their own interests.

Consider the following bargaining process between the vendor and the purchaser, as illustrated in Fig. 14: The vendor chooses an amount of the deposit d and announces it to the purchaser. Further, the vendor offers the purchaser a remanufacturing rate β with which the purchaser has to comply on the contractual basis.[1] She however retains the decision authority with regard to the order size. Assuming that she acts rationally and chooses an order size that minimizes her individual costs in response to the deposit amount and the remanufacturing rate

[1] We hereby assume that the vendor and the purchaser have agreed upon these bargaining terms in a preceding stage of negotiations.

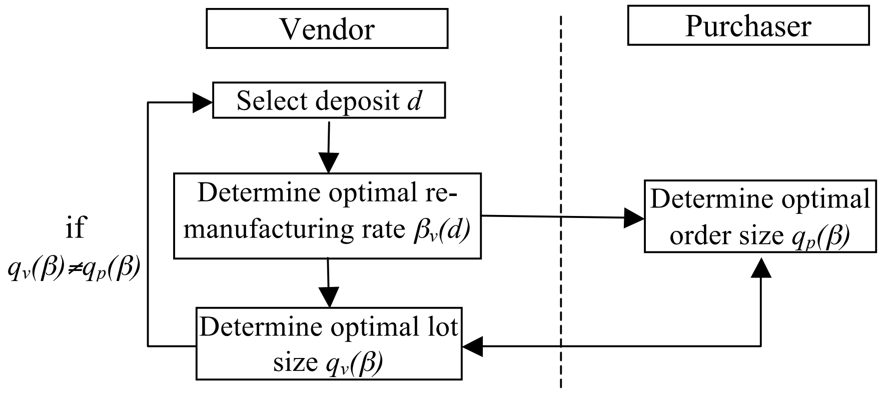

Fig. 14: The bargaining process

announced by the vendor, we henceforth address the following vendor's problem: what choice of the deposit and the remanufacturing rate is optimal for him? The problem under consideration thus represents a Stackelberg game with the vendor being the leader and the purchaser the follower; in more general terms, it represents a two-player extensive game with perfect information (Osborne and Rubinstein, 1994), a solution to which can be determined as a subgame perfect equilibrium by means of the backwards induction, as follows below.

By taking into account the purchaser's optimal response, the vendor is able to foresee the consequences of his decision (d, β) in terms of his resulting total costs — which we accordingly define as

$$\widetilde{TC}_v(d,\beta) \;=\; C_v(q_p^*(\beta),\beta) \;+\; R_v(d,\beta) \tag{50}$$

where $C_v(q,\beta)$ and $R_v(d,\beta)$ are vendor's cost components as defined by (5)–(6), and $q_p^*(\beta)$ is the purchaser's optimal order size as defined by (11). Note that we have now made $R_v(d,\beta)$ explicitly depend on d; also note that $q_p^*(\beta)$ does not depend on d. By substituting (5)–(6) and (11) in (50) we obtain:

$$\widetilde{TC}_v(d,\beta) = \frac{s_v D}{\sqrt{\dfrac{2s_p D}{h_p+\beta u_p}}} + \frac{\Delta_M\beta^2 - 2\Omega_M\beta + V}{2}\sqrt{\frac{2s_p D}{h_p+\beta u_p}} + (c_M + (d + c_R - c_M)\beta)D \tag{51}$$

The vendor's optimal choice of (d, β) thus represents a solution to the following constrained optimization problem:

$$\min_{(d,\beta)} \widetilde{TC}_v(d,\beta) \quad \text{s.t.} \quad d \ge 0,\ 0 \le \beta \le 1 \tag{52}$$

This problem must not necessarily be convex, therefore the Karush–Kuhn–Tucker (KKT) conditions (Bazaraa et al., 2006, p. 204) alone are not sufficient for determining a point of global optimum. However, they can be employed for determining potential local optima and subsequently choosing a global minimum out of them. To implement this approach, consider the Lagrange function of (52):

$$L(d,\beta,\lambda) \quad = \quad \widetilde{TC}_v(d,\beta) \quad + \quad \lambda\cdot(\beta-1) \tag{52}$$

whose arguments are nonnegative by assumption. The respective KKT conditions reduce to:

$$\frac{\partial L}{\partial d} = \beta D \geq 0 \qquad \text{and} \qquad d\cdot\frac{\partial L}{\partial d} \equiv d\beta D = 0 \tag{53}$$

$$\frac{\partial L}{\partial \beta} = \frac{\partial \widetilde{TC}_v}{\partial \beta} + \lambda \geq 0 \qquad \text{and} \qquad \beta\cdot\frac{\partial L}{\partial \beta} \equiv \beta\cdot\left(\frac{\partial \widetilde{TC}_v}{\partial \beta} + \lambda\right) = 0 \tag{54}$$

$$\frac{\partial L}{\partial \lambda} = \beta - 1 \leq 0 \qquad \text{and} \qquad \lambda\cdot\frac{\partial L}{\partial \lambda} \equiv \lambda\cdot(\beta-1) = 0 \tag{55}$$

An immediate consequence of condition (53) is the following

Lemma 8. There is no deposit refund as long as the vendor's optimal choice is concerned.

Indeed, the second part of (53) requires that either d or β or both is zero; if $\beta = 0$, no items are returned and hence there is no refund, otherwise $d = 0$ and therefore no refund, as well. The assertion of Lemma 8 is rather obvious: since the re-use rate is dictated by the vendor, the deposit has no relevance as an incentive for the purchaser to return used items, and refunding the deposit would represent just an unnecessary expenditure for the vendor.

Note that with $\beta = 0$, the choice of the deposit amount does not matter. We can therefore require $d = 0$ as a necessary optimality condition for all $\beta \in [0,1]$ and separate the subsequent analysis of (54)–(55) into three cases: $\beta = 0$, $0 < \beta < 1$, and $\beta = 1$, as follows.

a) $\beta = 0$. Then (55) implies $\lambda = 0$, and (54) is accordingly fulfilled whenever the derivative

$$\frac{\partial \widetilde{TC}_v(0,\beta)}{\partial \beta} = (c_R - c_M)D + \sqrt{\frac{2Ds_p}{h_p + \beta u_p}}\left(\varDelta_M\beta - \varOmega_M + \frac{u_p}{4}\left(\frac{s_v}{s_p} - \frac{\varDelta_M\beta^2 - 2\varOmega_M\beta + V}{h_p + \beta u_p}\right)\right) \tag{56}$$

is nonnegative at $\beta = 0$, i.e., whenever

$$(c_R - c_M)D + \sqrt{\frac{2Ds_p}{h_p}}\left(\frac{u_p}{4}\left(\frac{s_v}{s_p} - \frac{h_v}{h_p}\cdot\frac{D}{P_M}\right) - \varOmega_M\right) \quad \geq \quad 0\,. \tag{57}$$

Note that the radical in (57) represents the purchaser's classical economic order quantity.

b) $0 < \beta < 1$. As before, (55) implies $\lambda = 0$, while (54) effectively requires β to be a stationary point of $\widetilde{TC}_v(0, \cdot)$ — i.e., a solution to the equation that sets (56) equal to 0. The following examples show that such $\beta \in (0,1)$ must not necessarily exist, and when exists, can be a local minimum or a local maximum, as well.

Example 5a. Let $D = 400$, $s_v = 2000$, $s_p = 500$, $h_v = h_p = 50$, $u_v = u_p = 25$, $c_M = c_R = 50$, $P_M = 500$, $P_R = 2000$. Then

$$\widetilde{TC}_v(0,\beta) \approx 20000 + 1264.9\sqrt{25\beta + 50} + 316.2\frac{25\beta^2 - 10\beta + 40}{\sqrt{25\beta + 50}}$$

does not have any stationary points on $(0,1)$, as Fig. 15a demonstrates.

Example 5b. Let now $h_v = 100$. Then

$$\widetilde{TC}_v(0,\beta) \approx 20000 + 1264.9\sqrt{25\beta + 50} + 316.2\frac{55\beta^2 - 70\beta + 80}{\sqrt{25\beta + 50}}$$

has a stationary point at $\beta \approx 0.31$ which is a global minimum, see Fig. 15b.

Example 5c. Let further $P_M = 1250$ and $c_M = 57.5$. Then

$$\widetilde{TC}_v(0,\beta) \approx 23000 - 3000\beta + 1264.9\sqrt{25\beta + 50} + 316.2\frac{7\beta^2 + 26\beta + 32}{\sqrt{25\beta + 50}}$$

has a stationary point at $\beta \approx 0.18$ which is a global maximum, see Fig. 15c.

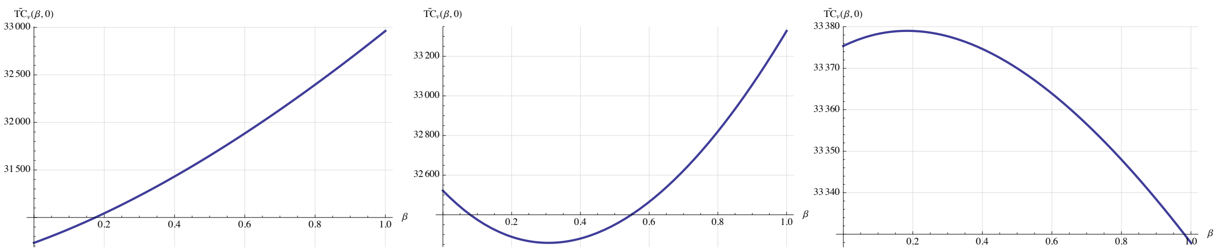

Fig. 15a–c: Vendor's cost function $\widetilde{TC}_v(0,\beta)$ in Examples 5a to 5c (left to right)

Although it is possible to obtain an analytic expression for a stationary point of $\widetilde{TC}_v(0,\beta)$, it turns out to be bulky and impractical to use; therefore analyzing the function's local extrema in each particular case would be a more suitable approach. Higher derivatives can help to reveal which stationary points represent local minima; however, this step is optional and can be replaced by explicit evaluation of the function value at all stationary points for subsequent comparison, as discussed below.

c) $\beta = 1$. Then (54) and (55) essentially require the function's derivative (56) to be no positive at $\beta = 1$, i.e.,

$$(c_R - c_M)D + \sqrt{\frac{2Ds_p}{h_p + u_p}}\left[u_v\left(1 - \frac{D}{P_R}\right) + \frac{u_p}{4}\left(\frac{s_v}{s_p} - \frac{2u_v + (h_v - u_v)D/P_R}{h_p + u_p}\right)\right] \le 0. \quad (58)$$

It would suffice now to compare the function value at each of the points determined in a) – c) to figure out a global minimum of $\widetilde{TC}_v(d,\beta)$.

Example 6. Let the data of Example 5c apply, and let $c = 3$. $\widetilde{TC}_v(0,\beta)$ has a positive (≈ 41.05) and a negative (≈ -109.24) derivative at $\beta = 0$ and $\beta = 1$, respectively, and a stationary point at $\beta \approx 0.18$ (see Fig. 15c). The function values at the three candidate points are 33375.36, 33327.92 and 33379.01, respectively, revealing that the global minimum is attained at $\beta = 1$. Computation of the function value at $\beta \approx 0.18$ is however not necessary in this case, since the derivative (56) is positive at 0 and negative at 1 while being continuous on [0, 1], therefore the only stationary point on (0, 1) can only be a maximum point.

With the vendor's optimal choice $(d,\beta) = (0,1)$, the purchaser's optimal response is the order size $q_p^*(1) \approx 73.03$ generating the costs of ≈ 5477.23 at the purchaser. Tab. 1 summarizes this optimal bargaining solution and compares it with the system-optimal one (cf. Section 4.3).

	d	β	q	TC_v	TC_p	TC_T
Bargaining	0	1	73.03	33 327.92	5 477.22	38 805.14
System optimum	0	1	119.52	30 577.77	5 477.22	36 733.2

Tab. 1: Comparison of bargaining and system-optimal solutions in Example 6

As the last column of Tab. 1 shows, the supply chain coordination is not achieved under this optimal bargaining solution; at the same time, the purchaser's costs are as low as they are under the jointly optimal solution (for which the zero deposit can be assumed, as well). It is worth noting at this point the analogy between the optimality of solution just obtained — being characterized by an extreme value of $\beta = 1$ which designates the pure remanufacturing strategy, as well as the solution emerging in Example 5a with $\beta = 0$ that designates the pure manufacturing — and the optimality of pure strategies established in Richter (1997).

As one can also see in the above example, the purchaser's choice of the order size under bargaining does not agree with the system-optimal order size. The vendor may attempt to choose such a deposit amount d which would — together with the vendor's respectively optimal remanufacturing rate $\beta^*(d)$ — induce the purchaser to choose an order size $q_p^*(\beta^*(d))$ equal to the order size $q_v^*(\beta^*(d))$ preferred by the vendor himself under this deposit amount d and remanufacturing rate $\beta^*(d)$. The following example demonstrates that when such vendor's choice exists, either, it still must not represent an optimal strategy.

Example 7. Let $D = 500$, $s_v = 300$, $s_p = 900$, $h_v = 50$, $h_p = 70$, $u_v = 5$, $u_p = 60$, $c_M = 20$, $c_R = 10$, $c = 3$, $P_M = 600$, $P_R = 2000$. Fig. 16 plots the respective order size functions $q_p^*(\beta^*(d))$ and $q_v^*(\beta^*(d))$ — where $\beta^*(d)$ is obtained by Corollary 2 and order sizes by equations (11) and (13), respectively. The plot shows that there is indeed a deposit amount $d \approx 13.15$ (with $\beta^*(d) \approx 0.31$) which makes the parties' individually optimal order sizes agree at ≈ 100.9. However, as Tab. 2 indicates, this solution is dominated by the optimal bargaining one — from both the vendor's and the entire system's perspective. The optimal bargaining solution closely approaches the system-optimal solution which is characterized by the remanufacturing rate of 1, order size of approx. 89 and the system-wide costs of approx. 18 472.19.

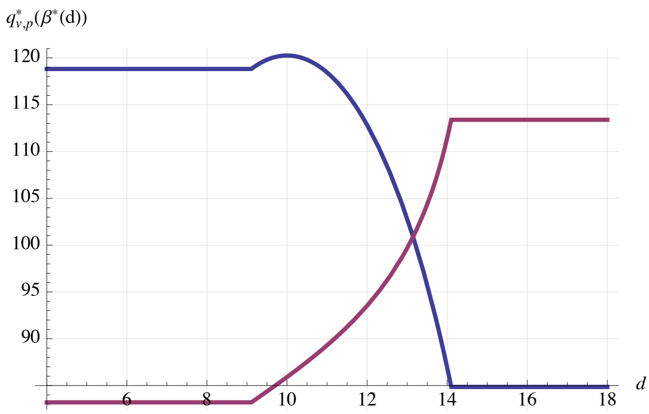

Fig. 16: Optimal lot sizes of the vendor (blue) and purchaser (red) as functions of the deposit amount d in Example 7

	d	β	q	TC_v	TC_p	TC_T
Equal order sizes	13.15	0.31	100.9	13 456.19	7 943.18	21 399.37
Optimal bargaining	0	1	83.2	7 686.83	10 816.65	18 503.48

Tab. 2: Comparison of the vendor's strategy inducing equal order size preferred by the parties with the optimal bargaining solution in Example 7

The following example demonstrates that such vendor's choice of the deposit amount d that would make the order sizes of the parties agree may not exist, either.

Example 8. Let the demand and the vendor data of Example 1 apply: $D = 100$, $s_v = 1000$, $h_v = 100$, $u_v = 5$, $c_M = 35$, $c_R = 20$, $P_M = 200$, $P_R = 250$, as well as the purchaser data of Example 4: $s_p = 400$, $h_p = 220$, $u_p = 40$, $c = 15$. Fig. 17 plots the respective order size functions of the vendor and the purchaser. As the lower plot in the figure demonstrates, the vendor has no way to entice the purchaser to choose the same order size he would himself prefer by adopting a certain amount of the deposit and sticking to the respectively optimal remanufacturing rate.

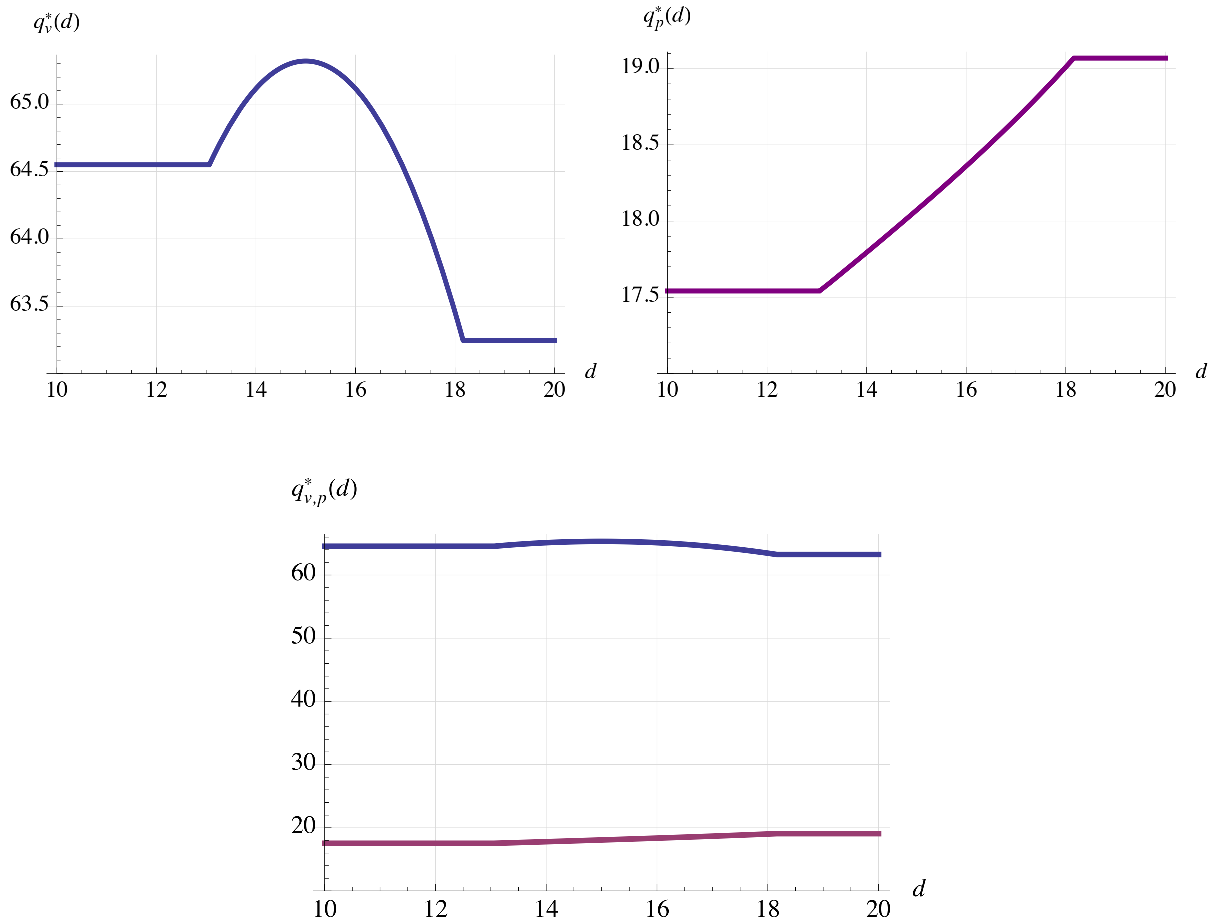

Fig. 15: Optimal lot sizes of the vendor (left) and purchaser (right) as functions of the deposit amount d in Example 5. The lower plot displays both functions on a common scale.

Conclusion

A complex of problems for determining individual and jointly optimal lot (order) sizes, remanufacturing (collection) rates and deposit amounts in a closed loop supply chain with a single suppler and a single buyer has been studied in the paper. While no analytic expression has been derived for the deposit amount under optimal bargaining, closed formulas have still been obtained for all other quantities. It has been demonstrated on examples that supply chain coordination must not necessarily be achieved in the bargaining game studied in the paper.

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