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Discussion Paper No. 270

May 2010

ISSN 1860 0921

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KEYWORDS: Matching market, price negotiation, optimal allocation, altruism

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* We would like to thank Yves Breitmoser and Werner Güth for their very helpful comments. This work has been funded by the *Deutsche Forschungsgemeinschaft* under the contract BO 747/11-1.

Abstract

The extant literature on matching markets assumes ordinal preferences for matches, while bargaining within matches is mostly excluded. Central for this paper, however, is the bargaining over joint profits from potential matches. We investigate, both theoretically and experimentally, a seemingly simple allocation task in a 2x2 market with repeated negotiations. More than 75% of the experimental allocations are unstable, and 40% of the matches are inefficient (in cases where inefficiency is possible). By defining the novel concept "altruistic core", we can explain the occurrence of inefficient matches as well as the significant behavioral differences among our six treatments.

1. Introduction

Imagine two small firms F_1 and F_2, both urgently in need of an accountant. The two accountants W_1 and W_2 available on the regional market both promise to do a good job but their productivities in terms of saved costs/additional income depend on the specific matches. Matches are defined as efficient if they maximize the sum of productivities. Will the market allocation be efficient? What kind of wages will the players negotiate? Will the matches be "stable", i.e. will no unmatched pair have the ability to make Pareto improvements by matching (the core condition)? In our experimental 2x2 market we observe that more than 75% of all matches are unstable. Theories such as Nash Bargaining or Shapley Value which are capable of making predictions outside of the core do not perform any better. They also predict efficient matches, although 40% of our observed matches are inefficient (in the cases where inefficient matches are possible), and in most cases of efficient matches their predictions lie around the edge of the cloud of points which describes the experimental results.

We attribute the weak performance of all these concepts to an overly-competitive attitude of the market participants. We will develop the concept of an "altruistic core" which offers a satisfactory explanation of our results under the assumption that, on average, market participants are spiteful. Negative altruism (spite) fosters the goal of receiving more income than one's partner does. Because these preferences may be restricted to market behavior we prefer to describe them as overly-competitive.

In the generalized problem there are F_i, i = 1, …, m and W_i, i = 1, …, n players on the two sides of the market. In many cases, these are *small number* markets, i.e. m and n are rather small. Internal job markets in firms is a good example, but also the assignment of jobs in academia, sports, or show business. Markets for top managers or for marriage partners in a remote village and also the competition for a small number of available locations in a shopping mall are further examples. In large markets, we may assume that the assignment problem is adequately described by a general search model. In small markets, however, we have to expect that all participants on one market side negotiate with all participants on the other side, resulting in strong strategic interactions.

In the marriage problem and similar frames, optimal and market matching are usually based on all participants' *ordinal rankings* of their potential partners. The question of stable and efficient matches is then investigated under different market institutions. (Marriage problem:

Gale and Shapley, 1962; Roth, 1984; Sasaki & Toda, 1992; Wolfstetter, 1996; Nosaka, 2007; Lundberg and Pollak, 2008; college admission problem: Roth, 1985; house or roommate allocations: Abdulkadiroglu & Sonmez, 1999; Kamecke, 1992; hospitals – new physicians: Roth, 1990; organ transplantation: Roth et al., 2004; law clerk matching: Haruvy et al., 2006). Many suggestions for matching mechanisms try to substitute “misleading” incentives which promote, for example, premature matches between hospitals and medical graduates, or between law clerks and Federal appellate judges. Optimal mechanism design by a centralized clearing house or similar measures is, however, not the focus of this paper.

For the assignment of workers or sites to firms it is usually assumed that every match has a certain productivity (in terms of money) which the partners have to split among themselves. For our investigation we will adopt this assumption of *transferable utility*. We want to find out which matches are formed in the market process and how productivities are split, i.e. in our experiments the partners in a match have to bargain about the distribution of their joint profit.

The literature on matching with transferable utility is rather limited. For the general case, Koopmans and Beckmann (1957)[1] show that market prices exist which support the efficient matches. Under these prices no other matches can be formed without making at least one of the partners of a potential other match worse off. The set of such prices is equivalent to the core (which is never empty in this problem). Becker (1974) investigates the marriage market under this and further simplifying assumptions (men and/or women are homogeneous). There are also some macro or intermediate approaches investigating the market efficiency under different labor market conditions such as the unemployment-vacancies structure and an information technology (i.e., Crawford and Knoer, 1981; Bolle, 1985; Hosios, 1990; Fujita & Ramey, 2006; Petrongolo & Pissarides, 2001, 2006; Fahr & Sunde, 2004; Sunde 2007).

There are only few experimental studies investigating matching markets. One example is Kagel and Roth (2000) who, in contrast to our study, do not allow their subjects to negotiate because they model a situation with ordinal rankings of partners. Their experiment reproduces a phenomenon found in many examples in the field, namely premature matches. An interesting result is reported by Haruvy and Ünver (2007), though again in a worker-firm environment with

[1] Shapley and Shubik (1971) independently came to the same conclusion.

ordinal rankings. They find no significant differences between high information environments (all players are completely informed) and low information environments (only their own ordering is known to them). Most similar to our experiment is Tenbrunsel et al. (1999) who, however, fully concentrate their investigation on the influence of personal relationships and the efficiency of resulting matches. General coalition experiments have been conducted by Albers (1986) and by Uhlich and Selten (1986), although with completely different payoff structures and explicit general bargaining among all group members (while our problem requires only pair-wise bargaining). Many other “matching” experiments consider buyer-seller relationships with homogenous goods where, in principle, all information about preferences can be comprised in one market price (see Cason and Noussair, 2007).

The next section derives some theoretical bargaining results for the 2x2 market concerning the core (= Neumann-Morgenstern solution in this case), Nash Bargaining, Shapley Value, and a concept which we call “Nash Bargaining with implicit threats”. Section 3 describes the 2x2 matching experiment with repeated negotiations. In Section 4, the results are presented and compared to the theoretical predictions. It will turn out that none of the above concepts are capable of explaining our experimental results. Section 5 investigates fairness and altruism considerations and proposes a new theory, the “altruistic core”, which explains why and when inefficient and unstable matches occur and how joint profits are split. Section 6 concludes the investigation.

2. Matching Theory

A match of W_i and F_j results in a productivity (joint profit) of $a_{ij}=w_i+f_j$ which W_i and F_j can distribute amongst themselves. w_i and f_j denote their respective payoffs. In the case of two workers and two firms we denote the productivities as

	Firm 1	Firm 2
Worker 1	α	β
Worker 2	γ	δ

Information

What is the informational status of the market participants? The "natural" assumption about information is that a worker knows the productivity of matches in which she may be involved (W_1 knows α and β) but not the productivities of matches of her competitor (γ and δ). The same applies for firms. In our experiments, we provided the subjects with exactly this type of information.[2]

For the following two solution concepts "core" and "Nash Bargaining with the outside option *no match*", the information about one's partner's income is sufficient. The "Shapley Value" and "Nash Bargaining with implicit threats (NBIT)" appear to require better information. It may be an interesting question whether the bargaining process makes the necessary information available, but we will not deal with this question directly. The weak performance of the Shapley Value and of NBIT may be due to several reasons, one of these being the lack of necessary information.

Core

Koopmans and Beckmann (1957) showed that there is a set of vectors (allocations) C = $\{(w_1, \dots w_m; f_1, \dots, f_n)$ with $w_i + f_j \geq a_{ij}$ for all i,j and $w_i + f_j = a_{ij}$ if (i, j) belongs to the optimal matches$\}$. C is equal to the core,[3] i.e. Koopmans and Beckmann (1957) showed that the core (defined later by Gillies, 1959) of the Matching Game is not empty. C is also equal to the unique Stable Set = Neumann-Morgenstern solution.[4] When matches A = $\{(W_1, F_1), (W_2, F_2)\}$ are efficient, i.e. $\alpha + \delta \geq \beta + \gamma$, the core solutions are called Core A are described by

(1) $w_1 + \delta - \beta \geq w_2 \geq w_1 + \gamma - \alpha$

(2) $w_1 + f_1 = \alpha,\ w_2 + r_2 = \delta,\ \ w_i, f_i \geq 0.$

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[2] Kagel and Roth (2000) have conducted their experiments under the same information structure.

[3] See Appendix A.

[4] See Appendix A. Note that in the case of ordinal preferences for partners there may be several Stable Sets, all containing the core (Ehlers, 2007).

If the matches B = {(W_1, F_2), (W_2, F_1)} are efficient, the core allocations are called Core B. (1) is reverted and (2) is reformulated accordingly.

In Figure 1 (at the end of this section) the core is illustrated. Outside the core there is exactly one alternative match which could be formed profitably. In our experiments, we often find that inefficient matches result. When A is efficient but B is chosen, (1) describes ”Anticore B” as the most “unstable” situation. Here all workers and firms could form more profitable matches. Outside Anticore B there is again only one match which could be formed profitably. One hypothesis is that, if the efficient matches are formed, we should find most allocations in the core. If inefficient matches are formed, we should find hardly any allocation in the anticore.

We do not discuss other set based concepts of Cooperative Game Theory. (Remember, however, our remark that Core and Stable Set coincide.) Instead, we introduce two “value approaches” with unique predictions (or at least smaller solution sets).

Shapley Value

The Shapley Value (Shapley, 1953)

(3) $$SV_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!\,(n-|S|-1)!}{n!}\left(v(S \cup \{i\}) - v(S)\right)$$

with a set N (of n players) and a value function $v(S) =$ maximal sum of productivities of all matches in S. $v(S)$ is often understood as a measure of “power”, in this case of bargaining power. For $\alpha \leq \beta \leq \gamma \leq \delta$ and if the matches A are optimal, the Shapley Values of Workers 1 and 2 are

(4) $$SV_{W1} = \frac{1}{3}\alpha + \frac{1}{6}\beta$$

(5) $$SV_{W2} = \frac{1}{6}\alpha - \frac{1}{3}\beta + \frac{1}{6}\gamma + \frac{1}{2}\delta.$$

If the matches B are optimal, then the Shapley Values of Workers 1 and 2 are

(6) $$SV_{W1} = \frac{1}{12}\alpha + \frac{5}{12}\beta + \frac{1}{4}\gamma - \frac{1}{4}\delta$$

(7) $$SV_{W2} = -\frac{1}{12}\alpha - \frac{1}{12}\beta + \frac{5}{12}\gamma + \frac{1}{4}\delta .$$

Note that the "asymmetry" of these values results from the relation $\alpha \le \beta \le \gamma \le \delta$. In Figure 1, the Shapley Values of Workers 1 and 2 are indicated as SV. They do not need to be in the core. If $\alpha + \delta = \gamma + \beta$ then (4) and (6) as well as (5) and (7) coincide.

Nash Bargaining Solution

We now determine the Nash Bargaining Solution and a variant of it which is adapted to the matching market. Let t_{W1}, t_{W2}, t_{F1}, t_{F2} be the threat values or "outside options" of workers and firms (which are to be determined later). The Nash Bargaining Solution results from the maximization of the Nash product

(8) $$P = (w_1 - t_{W1})(w_2 - t_{W2})(f_1 - t_{F1})(f_2 - t_{F2}) ,$$

here under the restriction that transfers are only possible within matches, i.e.

(9a) $w_1 + f_1 = \alpha$, $w_2 + f_2 = \delta$ or (9b) $w_1 + f_2 = \beta$, $w_2 + f_1 = \gamma$.

For $t_{W1} = t_{W2} = t_{F1} = t_{F2} = 0$ (threat = „no match") the result is

(10a) $f_1 = w_1 = \alpha/2$, $f_1 = w_1 = \delta/2$, or (10b) $w_1 = f_2 = \beta/2$, $w_2 = f_1 = \gamma/2$

In Figure 1 and Table 1 the combination with the larger Nash product is indicated as NB (T1, T2, and T3 matches A; T4, T5, and T6 matches B). For general threat values we find

(11a) $w_1^* = \frac{1}{2}(\alpha - t_{F1} + t_{W1})$ or (11b) $w_1^{**} = \frac{1}{2}(\beta - t_{F2} + t_{W1})$

and respective payoffs for the other players (see Appendix B).

Treatment	Productivities		NB (w_1, w_2)	SV (w_1, w_2)
T1	<u>280</u>	400	(140, 320)	(160, 300)
	400	<u>640</u>		
T2	<u>280</u>	280	(140, 320)	(140, 360)
	520	<u>640</u>		
T3	280	460	(140, 320)	(170, 290)
	460	640		
T4	160	<u>400</u>	(200, 260)	(150, 330)
	<u>520</u>	640		
T5	160	<u>460</u>	(230, 230)	(160, 300)
	<u>460</u>	640		
T6	280	400	(200, 260)	(160, 320)
	520	640		

Table 1: Productivities of matches (rows = workers, columns = firms, efficient matches are underlined). Nash Bargaining solution (NB), and Shapley Value (SV) of the six treatments as applied in our experiment. Note that in T3 and T6 both matching possibilities are efficient.

Assume that matches A result. The implicit threat t_{W1} of Worker 1 (who is in a match with Firm 1) is to offer Firm 2 a match (which results in the productivity β and) which makes Firm 2 indifferent, i.e. Worker 1 offers the profit $f_2^* = \beta - t_{W1}$. If matches B result, Worker 1 offers Firm 1 $f_1^{**} = \alpha - t_{W1}$. In the same way we can determine the other implicit threats (see Appendix B). We thus get a system of eight equations for the four threat values and the four payoffs. This system is linearly dependent but not contradictory. We therefore find only a linear condition for (w_1, w_2)

$$(12) \qquad w_2 = w_1 + \frac{1}{2}[\delta - \beta + \gamma - \alpha]$$

Outside options are smaller than zero if and only if (12) is restricted by

(13) $$\frac{\delta+\alpha-\gamma-\beta}{2} \leq w_1 \leq \frac{\alpha+\beta+\gamma-\delta}{2}.$$

(12) and (13) indicate all possible Nash Bargaining Solutions with implicit threats. In Figure 1 this line is indicated as NBIT. The matches A can only be formed if they are efficient. Otherwise $w_1^* < t_{W1}$; $w_2^* < t_{W2}$. With implicit threats, the Nash product is maximized with efficient matches (see Appendix B). Condition (12) remains unchanged if the matches B are formed and are efficient. In both cases it describes the middle of the restrictions (1). Condition (13) is substituted by

(14) $$\delta-\beta \leq w_1 \leq \frac{\alpha+\beta+\gamma-\delta}{2}$$

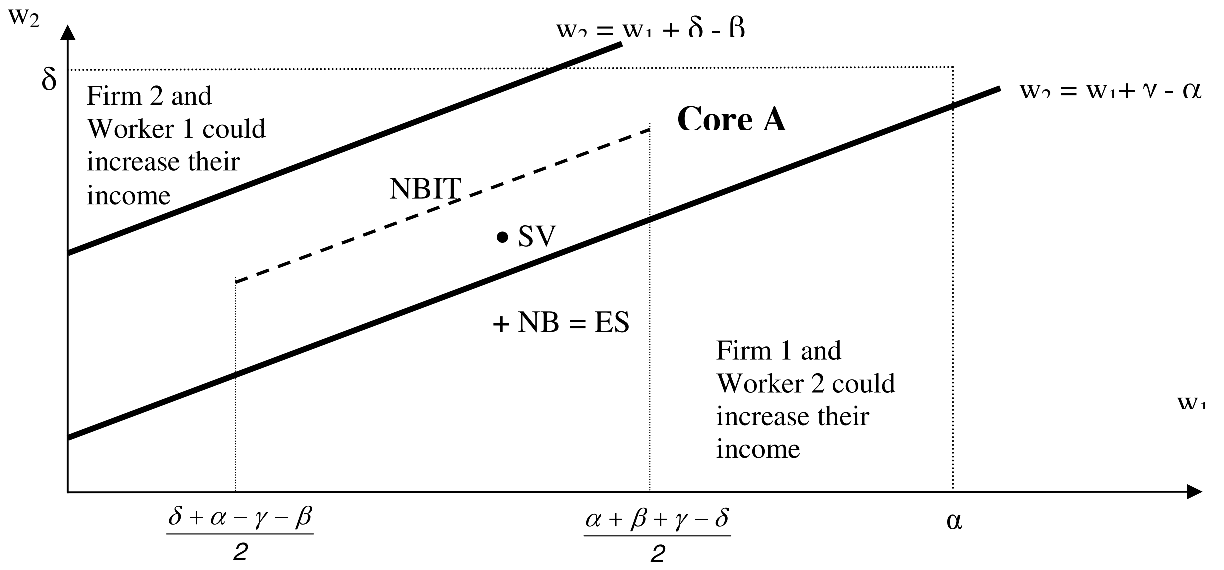
which is empty under the parameters of our treatment T4, but not so in treatments T5, T6.

Equal Split

As a very simple behavioral alternative we introduce ES = equal split of productivities in the matches chosen. I.e. ($w_1 = f_1 = \alpha/2$, $w_2 = f_2 = \delta/2$) for matches A and ($w_1 = f_2 = \beta/2$, $w_2 = f_1 = \gamma/2$) for matches B. In the case of efficient allocations in T1, T2, T4, and T5, NB and ES coincide.

Inefficiency

The above theories support only efficient results. Inefficient matches can result from boundedly rational or irrational behavior, from social preferences or, involuntarily, because other players stick to inefficient matches. Boundedly rational behavior may be detected by investigating the bargaining process in detail. In this paper, however, we want to concentrate on results. In Section 5, we will explain inefficient matches by social preferences, but first we want to confront the above purely strategic theories with the results of our experiment.



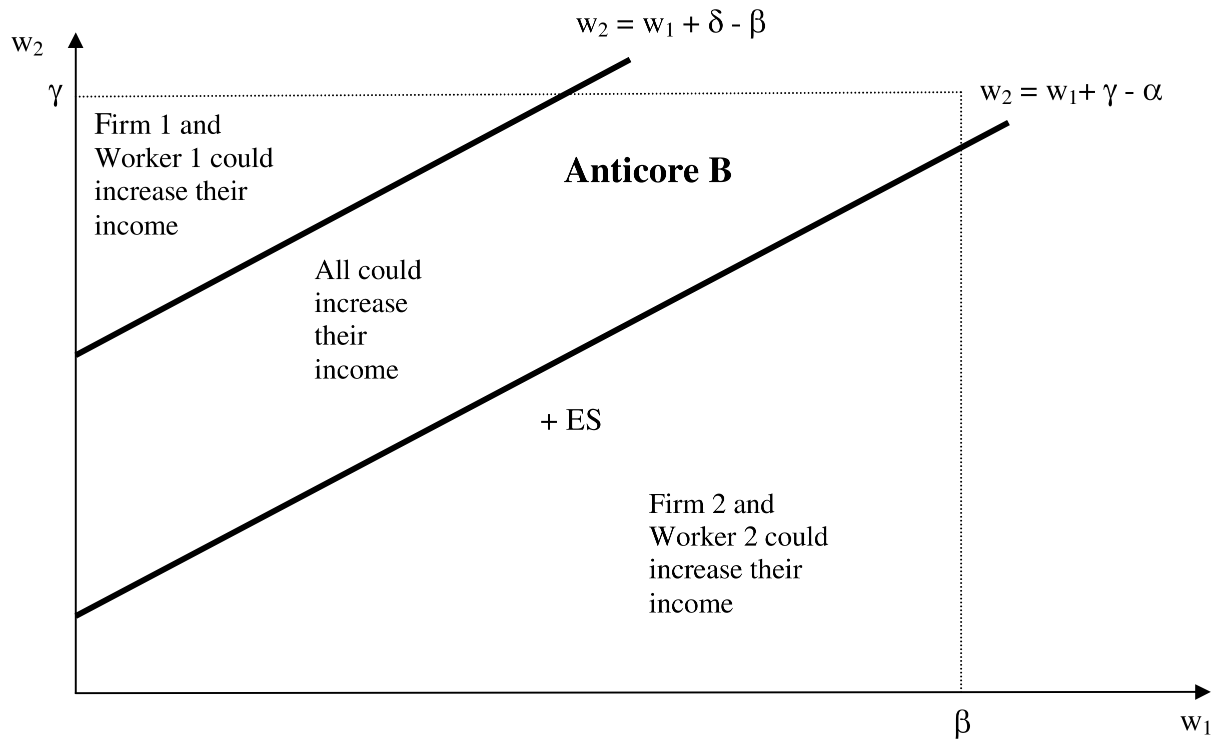


Figure 1: A is efficient and has occurred (top). A is efficient but B has occurred (bottom). SV = Shapley Value, NB = Nash Bargaining, NBIT = Nash Bargaining with Implicit Threats, ES = equal split. SV, NB, and ES can but need not lie in the core or in the anticore.

3. The Experiments

The two experiments simulate a simplified labor market situation with two workers and two firms and repeated one-on-one price negotiations. One experiment took place in a classroom setting with face to face interactions, and the other in a laboratory setting with anonymous computer based interactions. Face to face bargaining is a "more natural" situation while laboratory bargaining allows a better control of the experimental parameters.

In both experiments, in every session eight participants took part who were confronted with six different treatments (see Table 1). First, participants were separated into two groups of four subjects and assigned Worker 1 (W_1), Worker 2 (W_2), Firm 1 (F_1), or Firm 2 (F_2). The order of the treatments and the individual allocation to roles was randomized over the sessions. Every subject was allocated to a worker position three times and to a firm position three times. No subject was to interact with the same person more than three times. Every subject assumed each role W_1, W_2, F_1, and F_2 at least once. They took part either in the laboratory experiment (Lab) or in the classroom experiment (Class). Every session provided us with one independent observation.

In both experiments, participants drew their subject number which determined the sequence of their roles in the six treatments. The instructions were then handed out (for the different instructions in the laboratory and the classroom experiment see Appendix E). After remaining questions had been answered and everyone had confirmed their understanding of the task, the first matching phase began. In both experiments, the subjects were informed only about the productivities of the two matches they could participate in, i.e. W_1 was informed about α and β but not about γ and δ.[5] At the end of each matching phase the subjects were asked to evaluate their satisfaction and the fairness of the result, both on a five point scale.

The laboratory experiment (Lab) was run on z-Tree (Fischbacher, 2007). The individual allocation to a role was displayed on the start screen. On the next screen the matching process with submitting offers and accepting offers took place. An example screen display of this experiment is shown in Appendix D. During the negotiation phase, in the top box the individual

[5] We assumed this information structure to be "natural". Possibly it has affected the results considerably, though, in a matching experiment with ordinal preferences, Haruvy and Ünver (2007) do not find significant differences.

allocation was shown and the two boxes below were for the two partners. Here offers could be made or accepted. Once an offer was accepted, meaning a provisional contract had been made with this partner, this box became inactive and interaction was then possible only with the other potential partner. If a subject or her partner cancelled their contract by reaching an agreement with the other partner, new offers could be made and accepted. This phase continued until the allotted negotiation time of ten minutes for one treatment had expired. The last contract resulted in the individual payment for this treatment.

In the classroom experiment (Class), the firms of the two groups were seated at separate tables and were not allowed to move. They also wrote a protocol which contained every provisional match they formed together with the distribution of the respective profitability. Once an agreement had been settled, this was marked on the wall behind the table. The worker could now (if she wanted to and if there were no ongoing negotiations between the other two members of the group) start negotiations with the other firm. Negotiations were required to be one-on-one but it could not be prevented that there was some limited general communication. As in the laboratory experiment, a new agreement implied the cancellation of the old agreement. The negotiation phase of every treatment expired after ten minutes.

Why so many variations in our experiments? We believe that, in the case of bargaining, the question of whether face to face interactions yield different results than anonymous interactions is of particular interest. In important cases, bargaining *is* a face to face issue! Therefore we should try to find out which differences are caused by the laboratory situation which is usually preferred because it allows controlling for most parameters. In addition, we expected – and these expectations were confirmed – that the performance of bargaining theories might vary extraordinarily depending on the type of situation. In T1 and T2 efficiency requires matching with a partner who is in a similar strategic situation. In T4 and T5 efficiency requires matching between a strategically advantaged and a disadvantaged partner which should result in large income differentials within matches. In T3 and T6, where all matches are efficient, most theories give the subjects the choice of whether to match with an equal or unequal partner. This increase of discretion, however, causes the shrinking of the core from an area to a line. Requiring that a theory performs well under all these different conditions is a particularly strong test.

At the end of a session, the subjects answered several demographic questions before they received their income earned from all six treatments. In total, 160 subjects took part in the study.

80 students were randomly allocated to the laboratory experiment (average age 21.9, 47.5% male; 70% German; 25% Polish; 5% other nationalities) and 80 students were allocated to the classroom experiment (average age 22.8, 37.5% male; 70% German; 16.3% Polish; 14.8% other nationalities). In both experiments, ten sessions with eight participants took place. The total duration of the sessions was on average 1.5 hours in both experiments with an average payment of € 11.75 in the laboratory experiment and an average payment of € 13.41 in the classroom experiment.

4. Results

A graphical overview concerning matches and distributions of joint profits in the different treatments can be found in Appendix C. In Table 2, average results are reported for W_1 and W_2 without consideration of the cases of zero income if no match is formed. (In appendix C, such a case delivers a point on one of the axes.) From the relation $\alpha \leq \beta \leq \gamma \leq \delta$ we may derive strategic advantages resulting in $w_1 \leq f_1 < f_2 \leq w_2$. In general, the relation $w_1 < w_2$ holds for most of the cases (95.0%). The complete ordering $w_1 \leq f_1 < f_2 \leq w_2$, however, is found only in 30.5% of the cases. ($f_1 < f_2$ for 84.3% is smaller than the respective number in the case of the workers because W_2 is in a better position than F_2 in Treatments 2, 4, and 6). Our first conclusion is that strategic considerations do play a role.

Anonymous versus Face-to-Face Bargaining

There are hardly any differences between the laboratory and the classroom experiments with respect to the bargaining results in efficient matches or in inefficient matches (see Table 2). The most striking difference is the low number of incomplete matches (< 1%) in the classroom compared with the high number (17%) in the laboratory (see Table 3). This difference is highly significant (Fisher test; $p < .0001$). Among the matches formed in Treatments 1, 2, 4, and 5 there are also more inefficient matches in the Lab (46%) than in the Class (34%). This difference is weakly significant (Fisher test; $p = 0.051$). We conclude therefore that the direct contact and communication between subjects increases the efficiency in every respect, but it does not severely influence the average bargaining result in a given match. Thus, in most of the following analyses the results of Lab and Class are pooled.

The reason for the efficiency advantage of face to face bargaining may be twofold. On the one hand it is possible to communicate one's outside option or at least the profitability of one's alternative match. On the other hand there is simply more communication and exchange of arguments as well as the possibility of directly appealing to the interest in higher income or the fairness of one's negotiation partner.

			T1	T2	T3	T4	T5	T6
Lab	A	w_1	131	133	150	72	85	133
		w_2	340	327	340	326	331	334
	B	w_1	188	180	207	201	145	176
		w_2	222	270	263	347	245*	293
		$w_1 < w_2$	84.6%	100%	100%	91.7%	100%	100%
Class	A	w_1	160	143	145	80	83	149
		w_2	328	322	318	332	303	329
	B	w_1	188	158	199	167	200	160
		w_2	239	319	249	331	299*	324
		$w_1 < w_2$	90%	100%	94.7%	100%	100%	100%

Table 2: Average results for W_1 and W_2 differentiated according to treatment, to matches A or B, and laboratory or classroom experiment. Significant differences (on the <.05 level) between Class and Lab are indicated by an asterisk.

	T1		T2		T3		T4		T5		T6	
	Class	Lab	Class	Lab	Class	Lab	Class	Lab	Class	Lab	Class	Lab
No match (amount)	0	14	0	14	0	20	2	16	1	10	0	8
% of negotiations	0%	18%	0%	18%	0%	25%	3%	20%	1%	13%	0%	10%
A (amount)	**49**	**40**	**55**	**44**	**28**	**30**	23	34	52	54	**58**	**40**
% of matches	**61%**	**61%**	**69%**	**67%**	**35%**	**50%**	30%	53%	66%	77%	**72%**	**56%**
B (amount)	31	26	25	22	**52**	**30**	**55**	**30**	**27**	**16**	**22**	**32**
% of matches	39%	39%	31%	33%	**65%**	**50%**	**71%**	**47%**	**34%**	**23%**	**28%**	**44%**

Table 3: Absolute and relative frequencies of "no matches" and absolute and relative (with respect to matches) frequencies of A matches and B matches. Frequencies of efficient matches are in bold type.

Point Forecasts

The numerous cases of no matches and inefficient matches (see Table 3) cannot be explained by any of our strategic theories. Can these concepts explain at least the income distribution in the *efficient* matches? (Nash Bargaining even differentiates between efficient matches. It predicts A in T3 and B in T6.) The figures in Appendix C show, however, that in most cases the bargaining results are not centered around NB or SV. In the (efficient) matches B in T4, T5, and T6 78% of the w_1 results are smaller and 87% of the w_2 results are larger than NB predicts. If matches A are efficient then 75% of the w_1 results are smaller and 69% of the w_2 results are larger than SV predicts. Therefore, not even if we disregard the contradictions by inefficient matches do NB and SV provide us with a satisfactory description of behavior.

The Core and NBIT

As Appendix C and Table 4 show, the core is a successful predictive concept only in T1 (and perhaps in T3) and only if we concentrate on matches A. In all other cases of efficient matches (A in T2, B in T4 and T5) the number of results in the core is not higher than its relative area predicts (see Table 4). Selten's (1991) measure of predictive success for area theories is impressive only for the efficient choices of T1 (75%-18.75%=56.25%). In T1 and T2 there are, in the case of inefficient matches, no results in Anticore B (in accordance with our expectations). In T4 and T5, however, strategic considerations are foiled by the large numbers of bargaining results in Anticore A. These numbers are significantly higher than the relative area of Anticore A suggests. The tests will be discussed in more detail in connection with Table 6.

	A efficient				**B efficient**			
Treatment	**T1**		**T2**		**T4**		**T5**	
Experiment	Lab	Class	Lab	Class	Lab	Class	Lab	Class
(Anti-) Core A / Area A	18.75		18.75		18.75		18.75	
(Anti-) Core B / Area B	16.50		18.13		12.69		12.48	
Results in (Anti-) Core A	75*	84.62*	20	14.29	80*	100*	100*	71.43*
Results in (Anti-) Core B	0	0	0	0	14.29	21.43	16.67	23.08

Table 4: Results compared with predictions by the core. Area = percentage of points in the grid with a width of 1. * significantly ($p<.001$) higher proportion of choices within (Anti)Core according to a binomial test (two-sided).

In treatments T3 and T6 the core is only a line and difficult to evaluate with area theories. Nonetheless, the figures in Appendix C show that, in three of the four cases connected with T3 and T6, the results do not seem to be placed around the core. The same applies for NBIT which is a line in all treatments (except T4 where NBIT does not exist).

Learning

Our experiment was not established in order to investigate learning, in particular because the number of repetitions was only six and because every bargaining situation was different. In spite of this, we look for influences of the variable “period”, i.e. of the position of a treatment in the sequence presented to the subjects.

For efficient matches neither w_1 nor w_2 are significantly correlated with “period”. For inefficient matches, however, Table 5 shows significant developments. W_2 and F_2 learn to exploit their advantageous strategic positions. In addition, there is a trend ($r = 0.161$) toward efficient matches. This correlation coefficient is strongly significant ($p = .0013$).

	w_1		w_2		f_1		f_2	
	eff.	ineff.	eff.	ineff.	eff.	ineff.	eff.	ineff.
Correlation with period	-0.057	-0.380*	0.017	0.334**	0.043	-0.287*	-0.017	0.280*

Table 5: Trend or learning effects in efficient/inefficient matches. * (**) indicates significant correlation coefficients with $p < .05$ ($p < .01$).

Workers versus Firms

In treatments T1, T3, and T5, W_1 and F_1 as well as W_2 and F_2 are in the same strategic position. Do they earn the same amount? Yes, they do. In the laboratory as well as in the classroom, the differences are rather small and insignificant.

5. Altruism and/or Fairness

Though “… many authors have reached the conclusion that simple individual differences offer limited potential for predicting negotiation outcomes” (Bazerman et al., 2000), we find strong influence of individual fairness or altruism. After every matching game we asked the participants to evaluate the result in terms of “satisfaction” and “fairness”. We investigated whether their answers (on a five point scale) could be explained by altruism or inequity theories. Two prominent models highlight the influence of inequality in outcome satisfaction. First, Fehr and Schmidt (1999, henceforth F&S) suggest the utility function where it is assumed that $0<c_i$ and$0 \leq b_i \leq a_i$.

(15) $$U_i(x) = c_i x_i - a_i \frac{1}{n-1} \sum_{j \neq i} max\{x_j - x_i, 0\} - b_i \frac{1}{n-1} \sum_{j \neq i} max\{x_i - x_j, 0\},$$

Second, Bolton and Ockenfels (2000, henceforth B&O) assume

(16) $$U_i(x) = c_i x_i - a_i f\left(x_i \Big/ \sum_{j=1}^{n} x_j - \frac{1}{n} \right).$$

For empirical purposes we specified $f\left(x_i \Big/ \sum_{j=1}^{n} x_j - \frac{1}{n} \right) = \left| x_i \Big/ \sum_{j=1}^{n} x_j - \frac{1}{n} \right|^{\beta_i}$. In addition, as a simple alternative to inequity theories we consider altruism in the form

(17) $$U_i(x) = c_i + a_i x_i + b_i \sum_{j \neq i} x_j \ .$$

In all models the result depends on the question of whether person i considers the incomes of all four participants ("group") or only that of the person she is matched with ("match"). We computed both variants for all three models.

Empirical Results

The regression results (see Appendix F) for satisfaction[6] scores show that there is only a small *average* effect (rather small influences of altruism/inequity aversion) which seems to be captured equally well by all models (AdjR^2 between 0.2105 and 0.2256).

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[6] The R^2 values for the explanation of "fairness" were between 0.01 and 0.09 for all the three models which is considerably less than for satisfaction. Nonetheless it is debatable whether parameters of inequity theories should be estimated using both evaluations. This would have required, however, a lengthy discussion about methods.

The estimation of individual parameters is difficult because we have at most six observations for each individual. Therefore, we concentrated on the simplest model, namely the altruism model (17) which incorporates the smallest number of parameters. We chose the "match" version, because only in this case complete information is guaranteed. If the coefficient for i's own income (x_i) was $a_i \geq 0.001$ we determined $\eta_i = b_i / a_i$ as the normalized weight of others' income (x_j). If $a_i < 0.001$ we did not include this subject in our further investigation.

An indication that behavior was influenced by η_i is delivered by its connection with inefficient results. Figure 2 shows that the sign of η_i has a considerable impact on the number of inefficient matches a subject has agreed to. Spiteful subjects are involved in more inefficient matches than altruistic subjects. The correlation between results and altruism coefficients in Table 6, however, shows no apparent pattern. Two of the three significant coefficients are difficult to interpret. We think that the measurement of individual parameters is too imprecise to be used beyond their sign.

We conclude with a merging of strategic considerations and altruistic preferences. The accordingly derived "altruistic core" is capable of explaining the regularities of Figure 2 as well as most of those in the figures of Appendix C.

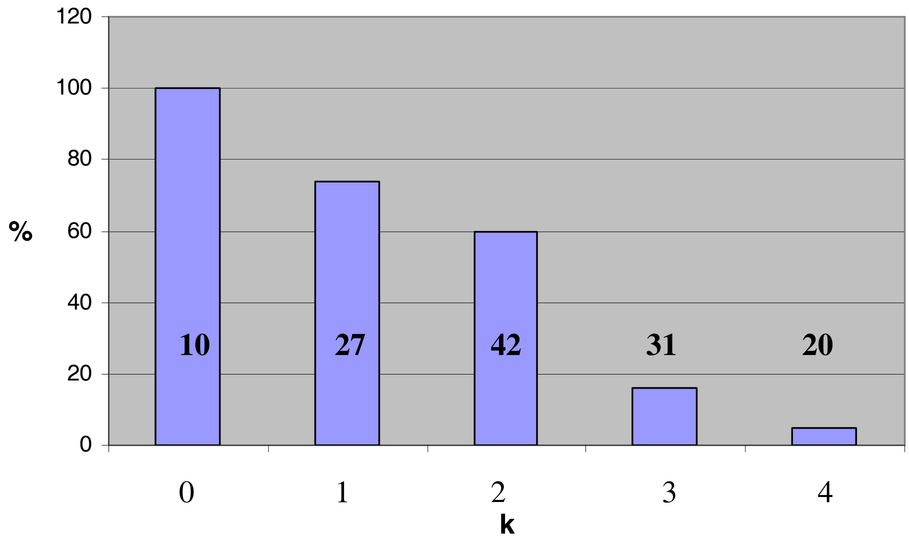

Figure 2: Percentage of participants with positive altruism coefficients depending on the number k of inefficient matches they formed (max = 4). Numbers in figure = number of cases.

		A-efficient				B-efficient				Both-efficient			
		T1		T2		T4		T5		T3		T6	
		A	B	A	B	A	B	A	B	A	B	A	B
w_1	own	-0.09	-0.21	-0.18	-0.33	-0.46	-0.20	0.07	-0.16	**0.53****	-0.38	0.37	-0.04
	other	0.37	-0.19	0.25	-0.38	0.17	0.03	-0.13	0.06	0.07	-0.45	-0.09	0.42
w_2	own	0.20	0.02	0.11	0.07	0.01	0.08	0.04	0.37	0.08	-0.26	0.15	-0.05
	other	-0.12	**0.48***	0.04	**-0.61***	-0.19	0.01	-0.2	0.09	0.15	0.19	-0.16	0.25

Table 6: Correlation between income and own/other's (in the match) altruism coefficient. (**) indicates significant correlation coefficients with $p < .1$ ($p < .05$).

The Altruistic Core

Let us assume that W_1 has an altruistic utility function (17) where she considers, in addition to her own income, only that of her partner in the match. If a match A results then

(18) $$\tilde{U}_{w1} = w_1 + a_{w1} f_1 = (1 - a_{w1}) w_1 + a_{w1} \alpha$$

or

(19) $$U_{w1} = w_1 + \frac{a_{w1}}{1 - a_{w1}} \alpha$$

describe W_1's preferences. Respective utility functions apply for W_2, F_1, and F_2. If matches B are formed then W_1 enjoys the utilities U'_{w1} where α is substituted by β, etc.

If the core (= set of non-dominated imputations) is connected with matches A, it is now characterized by allocations (w_1,w_2,f_1,f_2) with

(20) $$w_1 + f_1 = \alpha, \quad w_2 + f_2 = \delta$$

and

(21) $$U_{w1}' + U_{F2}' \leq U_{w1} + U_{F2} \quad \text{and} \quad U_{w2}' + U_{F1}' \leq U_{w2} + U_{F2}$$

for all (w_1',w_2',f_1',f_2') with $w_1' + f_2' = \beta$, $f_1' + w_2' = \gamma$. Substituting the utilities in (21) with (19) and with the respective equations provides

(22) $w_1 + \delta - \beta + \Delta_{12} \geq w_2 \geq w_1 + \gamma - \alpha + \Delta_{21}$

with $\Delta_{12} = \frac{a_{w1}}{1-a_{w1}}(\alpha - \beta) + \frac{a_{F2}}{1-a_{F2}}(\delta - \beta)$ and $\Delta_{21} = \frac{a_{w2}}{1-a_{w2}}(\gamma - \delta) + \frac{a_{F1}}{1-a_{F1}}(\gamma - \alpha)$

If matches B are formed, then

(23) $w_1 + \gamma - \alpha + \Delta_{11} \geq w_2 \geq w_1 + \delta - \beta + \Delta_{22}$

with $\Delta_{11} = \frac{a_{w1}}{1-a_{w1}}(\beta - \alpha) + \frac{a_{F1}}{1-a_{F1}}(\gamma - \alpha)$ and $\Delta_{22} = \left(\frac{a_{w2}}{1-a_{w2}}(\delta - \gamma) + \frac{a_{F2}}{1-a_{F2}}(\delta - \beta)\right)$.

Let us now evaluate "average" consequences of altruistic preferences. We do this by assuming all altruism coefficients to be equal, i.e. $\frac{a_h}{1-a_h} = \bar{a}$ for $h = W_1, W_2, F_1, F_2$. Under this assumption the core boundaries are moved in predictable ways. For matches A, the upper bound increases by $\Delta_{12} = (\alpha + \delta - 2\beta)\bar{a}$ and the lower bound by $\Delta_{21} = (2\gamma - \alpha - \delta)\bar{a}$. For matches B, the upper bound is moved by $\Delta_{11} = (\beta + \gamma - 2\alpha)\bar{a}$ and the lower bound by $\Delta_{22} = (2\delta - \gamma - \beta)\bar{a}$.

Interestingly, the altruistic core of 2x2 markets is fundamentally different for $\bar{a}$ above and below -0.5. From (22) and (23) and the values of Δ_{ik} it follows that, for $\bar{a} > -0.5$, the altruistic core requires efficient matches. If $\bar{a} < -0.5$, the altruistic core requires inefficient matches. If $\bar{a} = -0.5$, the altruistic core is a line and can be connected with efficient as well as with inefficient matches. Thus strong spite delivers an explanation for the occurrence of inefficient matches. The connection between *individual* spite and inefficient matches displayed in Figure 2 above is a first support of this thesis.

The bounds of the altruistic cores in the treatments of our experiment are shown in Table 7. If the lower bound is larger than the upper bound, then the respective altruistic core is empty. If there were average altruism $\bar{a} > 0$ among the participants in the 2x2 market, in most cases the consequence would be an altruistic core shifted upwards compared with the "egoistic" core. This contradicts the results shown in Appendix C. In T2, for example, the altruistic core for $\bar{a} = 0.5$ does not contain any experimental result.

The consequences of *average spite*, however, are mostly in line with these results. When inefficient matches are possible, i.e. in treatments T1, T2, T4, and T5, we separately determine the consequences of mild spite ($-0.5 \le \bar{a} \le 0$) and of strong spite ($-1 < \bar{a} \le -0.5$). On the basis of mild and strong *average* spite within a group, we expect (w_1,w_2) to lie in the union of all altruistic cores (UAC) connected with the respective $\bar{a}$ s (see Table 7). In T2 with matches A, for example, we expect (w_1,w_2) to lie between $w_2 = w_1+180$ and $w_2 = w_1+360$ ($\bar{a}$ between 0 and -0.5). In Table 8, the relative magnitude of the UACs (percentage with respect to all possible results) is compared with the hit rates, i.e. the percentage of results in the UACs. Note that in T3 and T6 all matches are efficient and altruistic cores exist for A and B matches and all $-1 < \bar{a} \le 0$. The UAC is defined as the union of all these altruistic cores. In 9 of the 12 cases there are significantly more results in the UAC than its area predicts. For the significant cases, Selten's (1991) measure of predictive success (= hit rate minus relative area of UAC) delivers high values except in the case of T3 with matches A. There, the altruistic core does not vary with $\bar{a}$. Thus the UAC is equal to a line (and equal to the "egoistic" core) which has a relative area (points in a grid with width of 1) of only 0.2%. The hit rate of 20% even in this degenerate case is due to the prominence of one point on the line. A closer look at the three insignificant cases shows that only T1 and T2 with matches B are rather unsatisfactory. The UAC of T6 with matches B is extremely large which prevents the high hit rate from being significant.

		T1	T2	T3	T4	T5	T6
A	**U**	**240+120$\bar{a}$**	**360+360$\bar{a}$**	180	240	180-120$\bar{a}$	240+120$\bar{a}$
	L	**120-120$\bar{a}$**	**240+120$\bar{a}$**	180	360+240$\bar{a}$	300+120$\bar{a}$	240+120$\bar{a}$
B	**U**	120+240$\bar{a}$	240+240$\bar{a}$	180+360$\bar{a}$	**360+600$\bar{a}$**	**300+600$\bar{a}$**	240+360$\bar{a}$
	L	240+480$\bar{a}$	360+480$\bar{a}$	180+360$\bar{a}$	**240+360$\bar{a}$**	**180+360$\bar{a}$**	240+360$\bar{a}$

Table 7: $w_2 = w_1+\ldots$ defines the upper (U) and lower (L) bounds of the altruistic core. In T1, T2, T4, and T5 cores with $-0.5 \le \bar{a} \le 0$ are connected with efficient matches (bold types) and cores with $-1 < \bar{a} \le -0.5$ are connected with inefficient matches. In T3 and T6, $-1 < \bar{a} \le 0$.

	T1		T2		T3		T4		T5		T6	
	A	**B**	**A**	**B**	**A**	**B**	**A**	**B**	**A**	**B**	**A**	**B**
N	21	12	24	9	24	10	10	21	11	19	23	13
Area UAC	18.8	42.0	28.1	41.2	0.2	62.9	18.8	43.8	18.8	37.9	18.8	62.3
Results in UAC	81.0*	33.3	66.7*	55.6	20.8*	100^{+}	60.0^{+}	95.2*	81.8*	100*	78.3*	84.6
Selten's Measure	62.2	-8.7	38.6	14.4	20.6	37.1	41.2	49.7	63.0	62.1	59.5	22.3

Table 8: Results compared with predictions of the UAC. Area = percentage of points in the grid with a width of 1. * $^{(+)}$ indicate significantly more results in the altruistic core than the area suggests according to a two-sided binomial test with p<.001 (p<.02).

The statistical tests in Table 8 as well as Table 4 have not taken into account the possible dependency of behaviour within a session. Within a session (at most) two results per treatment and match can occur. These can be interdependent because in every round (for every treatment) the eight participants are newly allocated to the two markets and the roles. Therefore, in an adjusted test, we substitute, in sessions with treatments where the two negotiations resulted in the same match A or B, the two data points by their average. In cases of one incomplete matching or in cases of different allocations A and B there is only one result. The tests on this basis are only marginally less powerful (here $^{+}$ indicates significance only on the 5% level, and in T2 A and T4 B the p-values are only 2% and 0.4%). Thus we still conclude that the altruistic core describes the bargaining results pretty well.

6. Conclusion and Discussion

The main conclusions from our experimental matching markets are:

(i) Bargaining results are influenced by strategic considerations, altruism (spite, an overly-competitive attitude), and learning (in inefficient matches).

(ii) Bargaining results are *not* influenced by the laboratory versus classroom situation, the worker versus firm role, or learning (in efficient matches).

(iii) Efficiency is affected by the laboratory versus classroom situation and average as well as individual altruism/spite.

Beyond these qualitative results we tested different bargaining theories. None of these (core=NM solution in our problem, Nash Bargaining, Shapley Value, as well as the self-developed “Nash bargaining with implicit threats”) is generally supported by our data. The merging of the core concept with altruism resulting in the “altruistic core” is successful, however. Assuming that on average spite dominates, the altruistic core explains the occurrence of inefficient and unstable matches (as a consequence of strong spite) as well as most qualitative differences among our treatments. Because these preferences may be restricted to bargaining behavior, we describe them as overly-competitive.

A possible variation of the 2x2 matching experiment is its generalization to larger markets. Similar to oligopoly experiments (Huck et al., 2004), an increasing number of competitors may reduce the deviation from the competitive equilibrium. Another question concerns the influence of complete information. In addition, a closer look into the dynamics of the bargaining process itself might provide insight. Here, we may find further explanations for non-matches and inefficiency as well as additional determinants of the resulting payoffs.

Our conclusion about the importance of social preferences is in line with most other experimental results concerning the interaction of a small number of agents. The dominating “flavor” of such preferences seems to depend, however, a lot on the situation. Bargaining seems to induce a strong competitive (spiteful) attitude in many people.

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Appendix A

C = core = unique Stable Set (von Neumann – Morgenstern solution).

First, let us derive a characteristic inequality. From $w_i + f_k \geq a_{ik}$ for all i, k follows that

$$\sum_{i \in B_W} w_i + f_{k(i)} \geq \sum_{i \in B_W} a_{ik(i)} \tag{24}$$

for all injective functions (= assignments) $k(i): B_w \to B_F$,

with $B_W \subset N_W = \{1,2,...,m\}$, $B_F \subset N_F = \{1,2,...,n\}, |B_w| < |B_F|$.

1. Now assume that a payoff vector (w_i, f_k) from C were dominated by another vector (from inside or from outside C) via the coalition (B_w, B_F). Without restriction of generality we can assume $|B_w| \leq |B_F|$, otherwise we exchange the roles of workers and firms. As all values stem form pair-wise assignments, the value of the coalition is described by $\sum_{i \in B_W} a_{ik^*(i)}$ with an optimal assignment $k^*(i)$. (24), however, shows that this value is too low to make the coalition better off than under any core allocation. Therefore all payoff vectors in C are undominated.

2. A payoff vector is outside C if one of the inequalities $w_i + f_k \geq a_{ik}$ is violated. Apparently this vector is dominated by every core allocation.

So C is equal to the core and it is also a stable set. Is it the only stable set? 1. Shows that we cannot remove any imputation from the core because it could not be dominated. 2. Shows that we cannot add any imputation to the core because it would be dominated. Therefore the core is equal to the unique Stable Set.

Appendix B

Nash Bargaining with implicit threats.

For general threat values we find

(25a) $w_1^* = \frac{1}{2}(\alpha - t_{F1} + t_{W1})$ or (25b) $w_1^{**} = \frac{1}{2}(\beta - t_{F2} + t_{W1})$

(26a) $w_2^* = \frac{1}{2}(\delta - t_{F2} + t_{W2})$ or (26b) $w_2^{**} = \frac{1}{2}(\gamma - t_{F1} + t_{W2})$

(27a) $f_1^* = \frac{1}{2}(\alpha - t_{W1} + t_{F1})$ or (27b) $f_1^{**} = \frac{1}{2}(\beta - t_{W2} + t_{F1})$

(28a) $f_2^* = \frac{1}{2}(\delta - t_{W2} + t_{F2})$ or (28b) $f_2^{**} = \frac{1}{2}(\gamma - t_{W1} + t_{F2})$

Assume that the matches A result. The implicit threat t_{W1} of Worker 1 (who is in a match with Firm 1) is to offer Firm 2 a match (which results in the productivity β and) which makes Firm 2 indifferent, i.e. Worker 1 offers the profit $f_2^* = \beta - t_{W1}$. If matches B result, Worker 1 offers Firm 1 $f_1^{**} = \alpha - t_{W1}$. In the same way we can determine the other implicit threats (see appendix).

(29a) $f_2^* = \beta - t_{W1}$ or (29b) $f_1^{**} = \alpha - t_{W1}$

The respective threats of Firm 1, Worker 2, and Firm 2 fulfill

(30a) $w_2^* = \gamma - t_{F1}$ or (30b) $w_1^{**} = \alpha - t_{F1}$

(31a) $f_1^* = \gamma - t_{W2}$ or (31b) $f_2^{**} = \delta - t_{W2}$

(32a) $w_1^* = \beta - t_{F2}$ or (32b) $w_2^{**} = \delta - t_{F2}$

Let us first regard the case where matches A are formed. Unfortunately, the system of the eight equations (25a) to (32a) is linearly dependent (but not contradictory). So we can determine only a linear condition for (w_1, w_2):

(33) $$w_2 = w_1 + \frac{1}{2}[\delta - \beta + \gamma - \alpha]$$

The respective outside options are

(34) $$t_{W1} = w_1 + \frac{1}{2}[\beta + \gamma - \alpha - \delta]$$

(35) $$t_{F1} = -w_1 + \frac{1}{2}[\alpha + \beta + \gamma - \delta]$$

(36) $$t_{W2} = w_1 + \gamma - \alpha$$

(37) $$t_{F2} = -w_1 + \beta .$$

The outside options t_{W1} and t_{F1} are not smaller than 0 if

(38) $$\frac{\delta + \alpha - \gamma - \beta}{2} \le w_1 \le \frac{\alpha + \beta + \gamma - \delta}{2} .$$

t_{W2}, $t_{F2} \ge 0$ is implied by (38). If the first inequality of (38) is not fulfilled, then we have to set $t_{W1} = 0$ which leads to $w_1 = \frac{\alpha + \delta - \beta - \gamma}{2}$. If the second inequality is not fulfilled, then we have to set $t_{F1} = 0$ which leads to $w_1 = \frac{\alpha+\beta+\gamma-\delta}{2}$. We do not analyze the case $\delta + \alpha - \gamma - \beta > \alpha + \beta + \gamma - \delta$ because, for the parameters in our experiment, this relation never occurs. Therefore (33) and (38) indicate all possible Nash Bargaining Solutions with implicit threats. In Figure 1 this line is indicated as NBIT. The matches A can only be formed if they are efficient. Otherwise $w_1^* < t_{W1}$; $w_2^* < t_{W2}$.

Is it possible that, with given outside options (threats) (20), (21), (22), (23), the Nash product is maximized with inefficient matches? Under the efficient match the Nash product consists of four equal factors $\frac{1}{2}[\alpha+\delta-\gamma-\beta]$. Thus, the Nash product is maximal under the given threats and under the profitabilities $\alpha + \delta > \beta + \gamma$. With inefficient matches, the Nash product must be smaller. T3 and T6 are degenerate cases but can be derived from $\alpha + \delta \rightarrow \beta + \gamma$.

Condition (33) remains unchanged if the matches B are formed and are efficient. In both cases it describes the middle of the restrictions (1). Condition (38) is substituted by

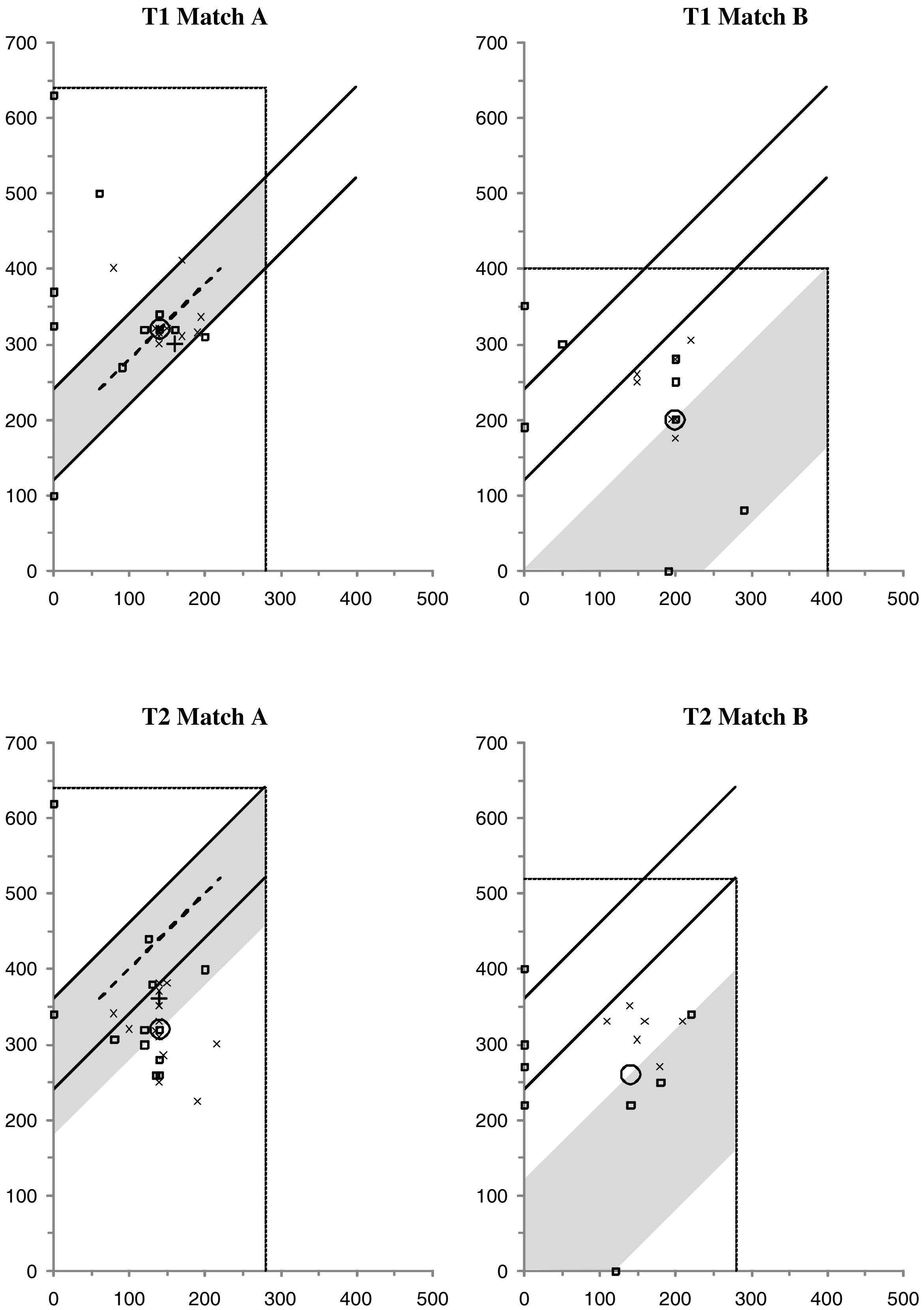
$$(39) \qquad \delta - \beta \leq w_1 \leq \frac{\alpha+\beta+\gamma-\delta}{2}$$

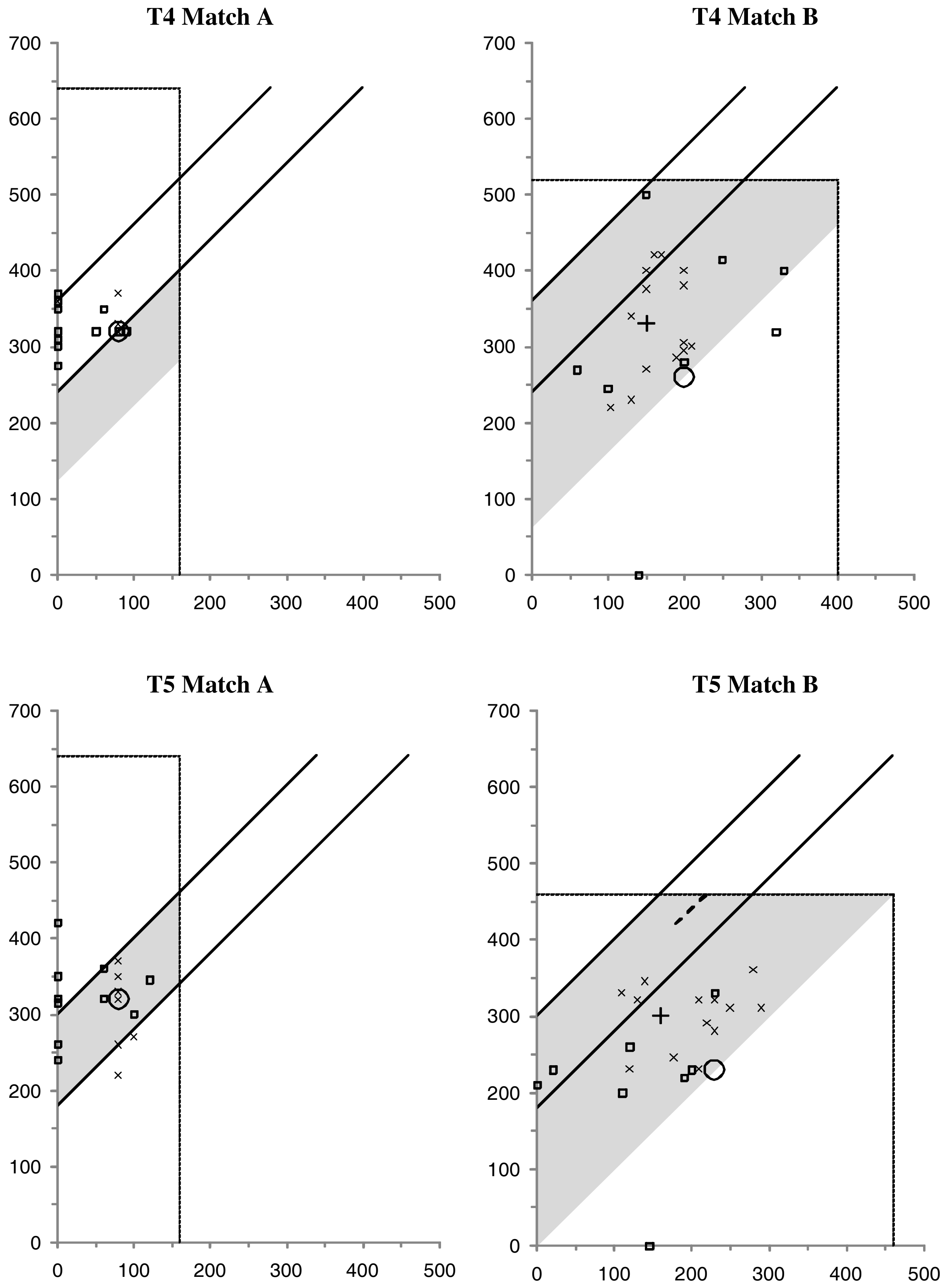
which is empty under the parameters of our treatment T4, but not so in treatments T5, T6.

Appendix C

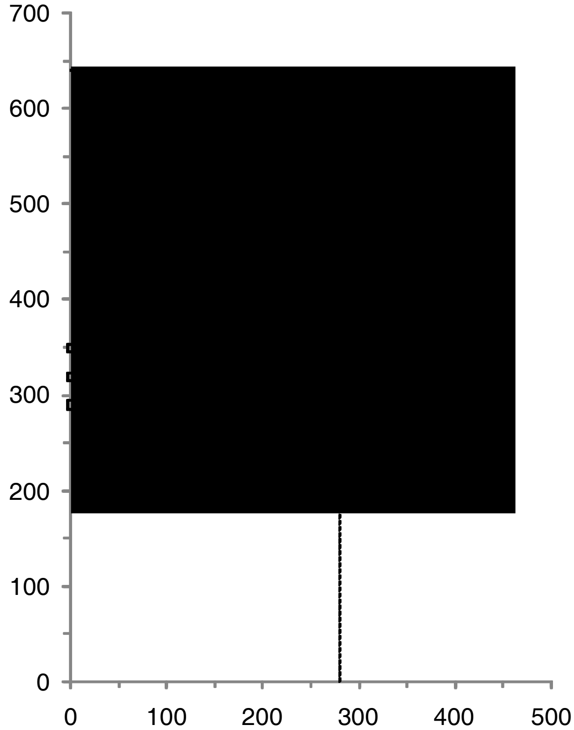
Results for Worker 1 (w1) and Worker 2 (w2). For illustration purposes the results are changed randomly by +/- 5 Cents. The legend is the same in all treatments/matches and the shaded region depicts the UAC. In T3 match A the altruistic core equals NBIT.

- o NB and/or ES
- + SV
- - - - NBIT
- ——— Core
- × Results Class
- □ Results Lab
- ······· Maximum

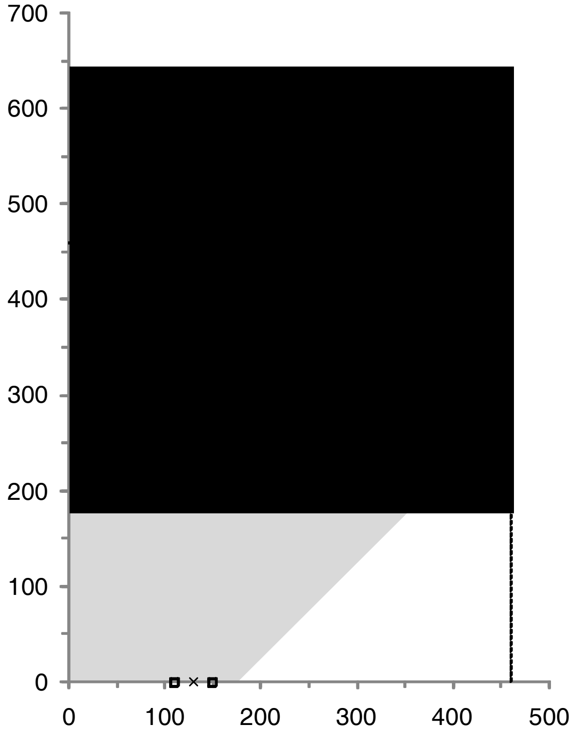

T1 Match A
T1 Match B
T2 Match A
T2 Match B
0
100
200
300
400
500
600
700
0
100
200
300
400
500


T4 Match A
T4 Match B
T5 Match A
T5 Match B
0
100
200
300
400
500
600
700

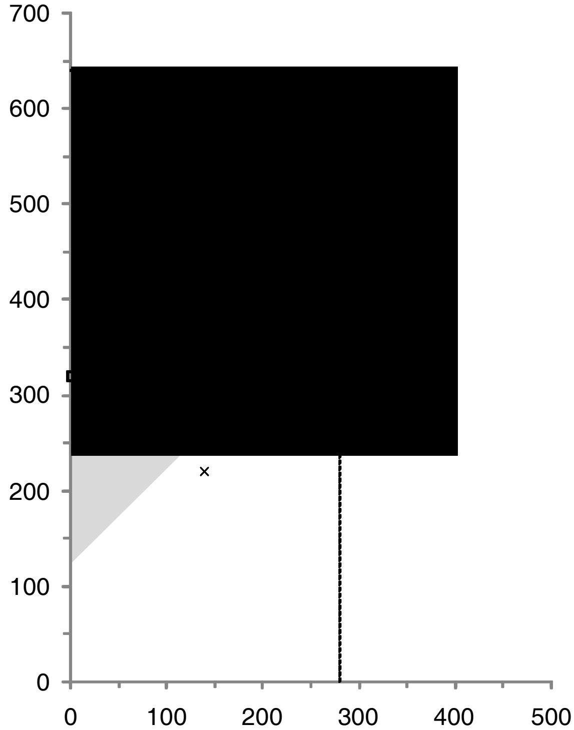
T3 Match A



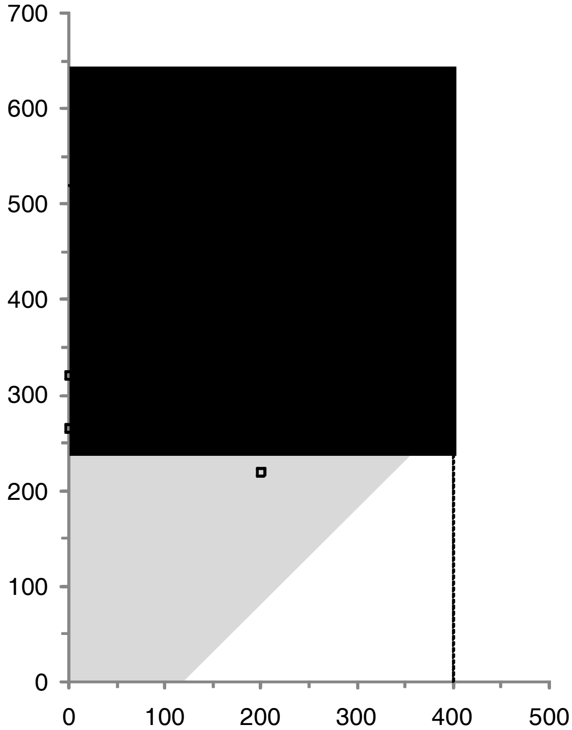
T3 Match B



T6 Match A

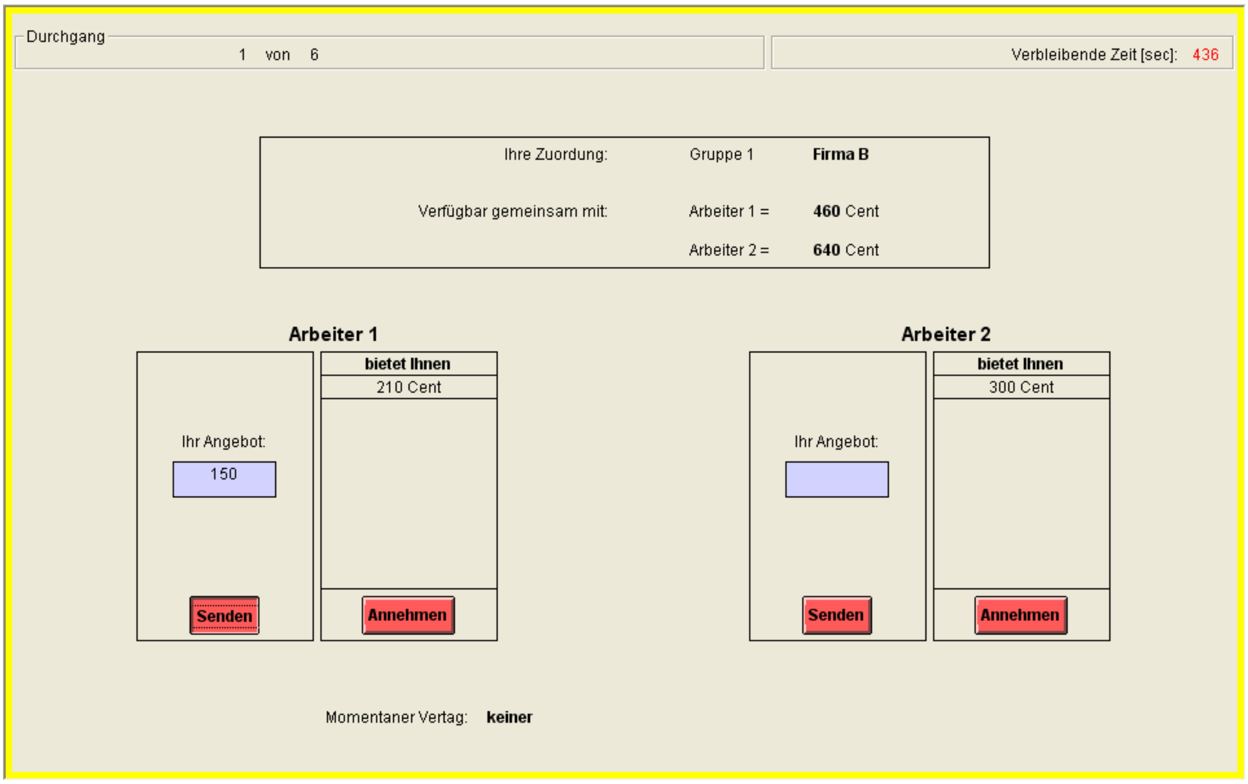


T6 Match B



Appendix D

Information provided at the negotiation phase of the laboratory experiment.



Appendix E

Experiment Instructions (translated from the original German version):

Thank you for participating in this labor market experiment. The experiment will last about one and a half hours. The payment you will receive at the end depends on both the decisions you make as well as on your co-players' decisions. The following provides an overview of the experiment procedure. Please notify the experimenter if you have any questions.

Our experimental labor market consists of two workers and two firms. Workers can be hired by (matched with) firms. Matches are only possible between one worker and one firm. Every worker-firm match earns a certain joint profit. In order for a match to form, the worker and the firm must first agree on the distribution of their joint profit.

We begin with the random allocation of the eight participants to two markets with each market consisting of two workers and two firms. You will receive a sheet of paper (Lab: see your computer screen) indicating whether you are a worker or a firm, as well as information about the different joint profits (in Eurocent) you would earn in a match with one of the two potential partners from the other market side.

Negotiations begin after workers have chosen a firm to bargain with. After one worker has made his choice, the other worker is allowed to bargain only with the remaining firm.

The players representing firms sit at tables and are approached by the workers. Only one-on-one negotiations are permitted. The first phase of the negotiation ends when a worker leaves the firm's table, regardless of whether a provisional agreement has been reached. In the case that an agreement is reached, firms are obliged to record this in a protocol. If both firm players are sitting alone at their tables, new negotiations may begin. This may lead to the cancellation of provisional agreements and the formation of new ones. The negotiation round expires after 10 minutes, at which point all provisional agreements become binding.

(Lab: Workers can send suggestions to firms to form a match with a certain joint profit distribution, and firms can send suggestions to workers. However, if a worker and a firm reach a provisional agreement, they are not allowed to bargain further until the provisional agreement expires. After 10 minutes, the negotiation round expires.)

After the first round, five additional rounds of negotiations all consisting of different market groups and different individual role allocations will ensue. At the end of the experiment you will receive the sum of all the shares of joint profits you have agreed to during the six rounds of negotiations.

Appendix F

Regression results for F&S are with $x_{more} = \sum_{j \neq i} \max\{x_i - x_j, 0\}$ and $x_{less} = \sum_{j \neq i} \max\{x_j - x_i, 0\}$:

F&S (group):
Satisfaction = 1.462 + 0.00916 x_i - 0.00389 (x_{more}) - 0.0000508 (x_{less}) [AdjR^2= 0.2256]

F&S (match):
Satisfaction = 1.864 + 0.00622 x_i - 0.000339 (x_{more}) - 0.000343 (x_{less}) [AdjR^2= 0.2105]

Regression results for B&O are with $x_{rel} = \left| x_i \middle/ \sum_{j=1}^{n} x_j - \frac{1}{n} \right|$. The exponential parameter of B&O is varied from 0.1 till 2.5 in steps of 0.1.

B&O (group):
Satisfaction = 1.874 + 0.00644 x_i - 4.761 $(x_{rel})^{1.8}$ [AdjR^2=0.2139]

B&O (match):
Satisfaction = 1.972 + 0.00625 x_i - 0.251 $(x_{rel})^{0.1}$ [AdjR^2=0.2132)]

Several studies (e.g. Engelmann & Strobel, 2004) show that the separation into two different effects, in line with F&S, better describe experimental results. But the regression results here show no major differences between the two fairness models. The results for the altruism model are (with $x_{other} = \sum_{j \neq i} x_j$):

Altruism (group):
Satisfaction = 1.53453 + 0.006314x_i + 0.0009580 x_{other} [AdjR^2= 0.2199]

Altruism (match):
Satisfaction = 1.85186 + 0.006193x_i - 0.00003834 x_{other} [AdjR^2= 0.2108]

The parameter of the own result x_i is always significant ($p < .001$ for all models). In addition, x_{more} in F&S (group) and x_{rel} in B&O (group) as well as in B&O (match) are significant (p = .0003; p = .0199; p = .0269). For the Altruism model the parameter x_{other} is significant in Altruism (group) with p = .0015.