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Abstract

Measurement in financial accounting often requires determining an interest rate to discount future cash flows. One example is the International Accounting Standard (IAS) 36 Impairment of assets. IAS 36's impairment test requires determining a value in use (a present value). The Appendix A to the standard gives some guidance on how to determine a suitable discount rate. In this paper, we show that the different approaches included in IAS 36's guidance are theoretically different. We discuss how the standard should be interpreted and applied based on the theoretical background of financial theory. Only the first alternative, the weighted cost of capital should be used and the other two alternatives should be discarded. In addition, we show that IAS 36's guidance, applied in practice, may give rise to substantial measurement errors.

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1 Introduction

Present values are frequently used in financial reporting under International Financial Reporting Standards (IFRS). For example, IAS 36's impairment test requires the reporting entity to determine a value in use. The value in use is defined as ,,the present value of the future cash flows expected to be derived from an asset or cash-generating unit“.[1] According to IAS 36.55, the discount rate should reflect the time value of money and the risks specific to the asset, as far as the future cash flow estimates have not been adjusted for these risks. One major difference between IAS 36's present value (value in use) and the present values used in the common discounted cash flow (DCF)-models lies in the requirement to use pre-tax amounts for both the future cash flows to be discounted and the discount rate.[2]

IAS 36's appendix gives comprehensive guidance on how to determine the appropriate discount rate (commonly called ,,cost of capital“ in finance theory). In general, the cost of capital may be estimated by comparison to market prices of similar assets and adjusted if appropriate to reflect the specific risk associated with the assets.[3] Appendix A.17 (to IAS 36.57) goes on stating: ,,As a starting point in making such an estimate, the entity might take into account the following rates:

- the entity's weighted average cost of capital determined using techniques such as the Capital Asset Pricing Model;
- the entity's incremental borrowing rate;
- and other market borrowing rates.“

The wording implies a choice at the discretion of the reporting entity. This gives the impression that either the weighted average cost of capital (WACC) or the incremental borrowing rate can be used as alternative starting points for determining an appropriate discount rate.

The aim of this paper is to show and to analyze the functional interrelation between the WACC, the cost of borrowing and the incremental borrowing rate and to discuss the implications of these interrelations for IAS 36's value in use.

1. We will show that the two ,,starting points“ mentioned above in fact represent completely different concepts. The functional interrelation between them is very complex even under the simplest assumptions.

[1]See IAS 36.6.

[2]See for example *Brealey and Myers* (2003), chapter 19 for a discussion of the various DCF-models.

[3]See IAS 36.56 read in conjunction with appendix A15 and A16. However, costs of capital readily observable in the capital market will exist only rarely, as the former International Accounting Standards Committee (IASC) acknowledges, see IAS 36.BCZ 55.

According to finance theory, the WACC is the only appropriate mean for determining a value in use. Consequently, alternatives (2) and (3) in IAS 36.55 should be discarded. Even if the theoretical differences between the WACC and the incremental borrowing rate are left aside, the errors in estimating the cost of capital can be substantial for highly leveraged reporting entities. We will show that if the incremental borrowing rate is used by a highly leveraged entity, the determination of the value in use will be erroneous: The value in use will be systematically too low.

2. In addition, we will demonstrate that the interpretation of the term ,,incremental borrowing rate“ gives rise to the possibility of earnings management: The incremental borrowing rate for capital will differ significantly if lending is risky, depending on whether the claims will be satisfied

   - after any other claims (subordinated debt, junior debt) or
   - together with other claims (pari passu, senior debt)

   on liquidation.

2 Assumptions and Notation

Our analysis of the functional interrelations between the WACC, the cost of borrowing and the incremental borrowing rate is based on the following assumptions:

1. Markets are frictionless, i. e. there are no transaction costs, bankruptcy costs, taxes, or restrictions on short selling. Information is costless and simultaneously available to all investors.
2. Operating cash flows are independent of the choice of capital structure; there are no business disruption costs.
3. Investors maximize their end-of-period wealth. They have homogeneous expectations about asset returns; the instantaneous rate of return on any asset and the market portfolio have a normal distribution. Investors may borrow or lend unlimited amounts at the risk-free rate.

The following notation is used in this paper:

$\widetilde{CF}$	Cash flow after investing, before interest, amortization and taxes
CU	Currency Unit

Z	Interest
T	Amortization
BV	Book value of the entity's debt
D	Market value of the entity's debt
E^u	Market value of the unleveraged entity's equity
E^l	Market value of the leveraged entity's equity
V^u	Market value of the unleveraged entity
V^l	Market value of the leveraged entity
k_D	Borrowing cost
k_E^l	Cost of equity of the leveraged entity
k_E^u	Cost of equity of the unleveraged entity
r_f	Risk-free interest rate
i	Nominal borrowing rate for the entity's debt
i_n	Nominal borrowing rate for subordinated debt
$WACC$	Weighted cost of capital
σ	Volatility of normally distributed yields
μ	Expectation of normally distributed yields
$N(.)$	Standard normal distribution

3 The Modigliani-Miller-Model

Modigliani and Miller (1958) showed that a firm's market value is independent from its capital structure (its leverage):[4] The market value of an leveraged entity V^l equals the value of the unleveraged entity V^u,

$$V^l = V^u . \tag{1}$$

Stiglitz (1969) und *Rubinstein* (1973) showed that the capital structure is irrelevant even if debt is risky. This generalization of the Modigliani-Miller-Model (1) may be proved by means of some simple lines of thought based on arbitrage: On a perfect market the leveraged entity's market value V^l equals the sum of the market value of the leveraged entity's equity E^l and the market value of the entity's debt D,

$$V^l = E^l + D. \tag{2}$$

[4]The proof essentially bases on assumptions 1 und 2, see chapter 2.

The claims of the equity holders of an unleveraged entity would equal the combined equity holders' claims and debt holders' claims if the entity was leveraged:

	Payoffs at Maturity	Current Value
Shareholder's position	$\max(\widetilde{CF} - Z - T, 0)$	E^l
Bondholder's position	$\min(Z + T, \widetilde{CF})$	D
Sum of the positions	$\widetilde{CF}$	V^l

Consequently, the Modigliani-Miller-Model (1) holds true also for risky debt. This model is of the utmost importance for the explanation of different costs of capital. In general, the cost of capital of any entity may be defined as the expected yields of the respective groups of claimants,

$$k_E^l \;=\; E\left[\max(\widetilde{CF} - Z - T, 0)\right] / E^l - 1\,, \tag{3}$$

$$k_D \;=\; E\left[\min(Z + T, \widetilde{CF})\right] / D - 1\,, \tag{4}$$

$$WACC \;=\; E\,[\widetilde{CF}]\,/\,V^l - 1\,. \tag{5}$$

From some transformations of the definitions (3), (4) and (5), follows

$$WACC \;=\; k_E^l\, E^l / V^l + k_D\, D / V^l\,. \tag{6}$$

This is a common definition for WACC. If the cost of capital of an unleveraged entity be defined as

$$k_E^u \;=\; E\,[\widetilde{CF}]\,/\,V^u - 1\,, \tag{7}$$

then, following directly from the Modigliani-Miller-Model (1) and the definition of the costs of capital (5) it can be shown that WACC is independent of the debt-equity-ratio and equals the cost of equity of an unleveraged entity,

$$WACC \;=\; k_E^u\,. \tag{8}$$

Furthermore, it can be shown, deriving from (6) and (8), that the cost of equity is a function of the entity's debt-equity-ratio D/E^l,

$$k_E^l \;=\; k_E^u + (k_E^u - k_D)\, D/E^l. \tag{9}$$

If lending/borrowing is not risky, this is a linear function of the entity's debt-equity-ratio.

Figure 1 illustrates the implications of the Modigliani-Miller-Model (1) for the relation between the entity's cost of capital: The WACC equal the cost of equity of the unleveraged entity, independent of the debt-equity-ratio. For low debt-equity-ratios the cost of equity increases linearly with the entity's debt-equity-ratio. If the debt-equity-ratio reaches the level where the debt becomes risky, the borrowing costs will rise and the slope of the cost of equity will decrease.[5]

[5] In table 1, this is the case when the debt-equity ratio D/E^l is approximately 2.

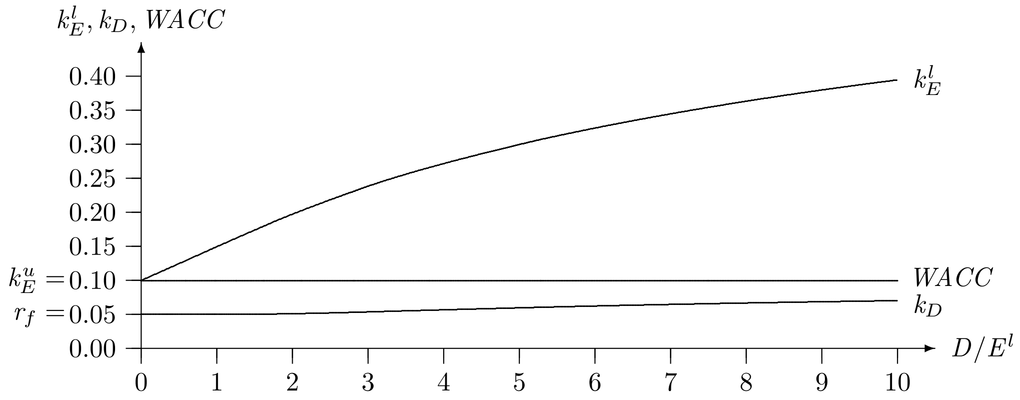

Figure 1: Cost of Capital of a Leveraged Entity

In chapter 4, our first step is to explain the interrelation between the market value of debt and equity based on the option pricing theory. Afterwards, in chapter 5, we analyze the interrelation between the cost of capital and the nominal borrowing rate when debt is risky. Finally, in chapter (6), we will determine the incremental borrowing rate.

4 Equity as a Call Option and the Value of Debt

In order to explain the relation between the market value of the entity's equity and its debt, assuming given and fixed investing activities, we will go back to the Modigliani-Miller-Model (1) in connection with the no-arbitrage-restriction (2),

$$V^u = E^l + D. \tag{10}$$

As shown above, this equation is valid also if debt is risky, however, it does not explain the relation between the market value of the entity's equity and the market value of its debt. It does also not explain if this relation is a function of the entity's debt-equity-ratio. If so, it does not explain what kind of function it is. The relation of the equity's and debt's market values may be explained either by the Capital Asset Pricing Model (CAPM) or the option pricing theory. Both are consistent with one another.[6] We will use the option pricing theory, because it allows us a preference-free determination of the values.

Black and Scholes (1973) were the first to show that the equity of a limited liability company may be viewed as a call option: The owners of the company will only claim the company's cash flows if the cash flows exceed the amortization and interest to be paid. If not, they will forfeit the option and ask for liquidation of the company in accordance with the local insolvency proceedings. Consequently, the cash flows to the equity holders of a

[6] *Hsia* (1981) proved the consistency of the Modigliani-Miller-Model, the option pricing theory and the CAPM.

leveraged entity are the same as the cash flows resulting from a call on an unleveraged, but apart from that, similar entity. The call's strike price equals the sum of interest and amortization to be paid by the leveraged entity.

In a similar way, *Merton* (1974) showed that the claims of the creditors may be regarded as the interest plus amortization less the pay-off of a put (see figure 2). Therefore, the market value of the risky debt will decrease by the market value of a put, which could completely compensate the loss of the creditors in case of bankruptcy.

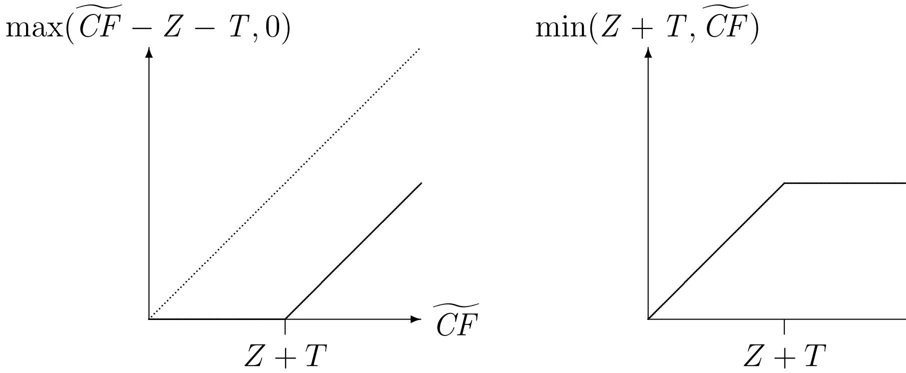

Figure 2: Shareholders' and Bondholders' Payoff at Maturity

Based on that, the relation of the market values of equity and risky debt may be explained using option pricing theory. Using the Modigliani-Miller-equation (10) it can be shown that

$$V^u = \underbrace{\text{Value of Call}\,(Z+T)}_{=E^l} + \underbrace{(Z+T)/(1+r_f) - \text{Value of Put}\,(Z+T)}_{=D}, \quad (11)$$

which is basically the common put-call-parity. Starting from the market value of an unleveraged entity, we are now able to explain the relation between the market values of equity and debt and an increasing debt-equity-ratio e.g. with the model of *Black and Scholes* (1973).

Exhibit 1 *Assume the market value of the unleveraged entity V^u with an entity's life of one year is 100 CU, the volatility of the normally distributed yields is $\sigma = 20\,\%$ p. a. and the risk-free interest rate is $r_f = 5\,\%$ p. a. If the equity holders decide to substitute part of their equity with debt, the market value of the leveraged entity V^l will not change, according to the Modigliani-Miller-Model (1). Assuming contractually fixed amortization and interest is e. g. 80 CU, it follows, according to* Black and Scholes *(1973) and the put-call-parity (11), that the market value of the equity will be 24.51 CU, the market value of the debt will be 75.49 CU.*

5 The Cost of Debt and the Promised Yield on Debt

The market value of the entity's debt equals, according to (4), the present value of the creditors, that is, the creditor's expected cash flows, discounted with the cost of debt k_D (borrowing cost). The creditors calculate the nominal borrowing rate i according to the market value of their debt. The amortization and interest $(Z + T)$ which has to be contractually paid by the entity, discounted with the nominal borrowing rate, will also equal the market value of the debt on an capital market, assuming it is free of arbitrage,

$$D = E\left[\min(Z + T, \widetilde{CF})\right] / (1 + k_D) \;=\; (Z + T) / (1 + i)\,. \tag{12}$$

Thus, when the debt is not risky, the cost of debt k_D equals the contractual nominal borrowing rate i. However, if debt is risky, the cost of debt will always be smaller than the nominal borrowing rate $(k_D < i)$. Based on the equations for the market values of debt and equity we derived in chapter 4, we are now able to determine the cost of capital (3), (4) and (5) and the respective nominal borrowing rates, provided the expected cash flows of both creditors and equity holders are known. It seems that this relation has been somewhat unnoticed so in the literature so far. [7]

Exhibit 2 *All data is the same as in exhibit 1. In addition, let the expected yield of the cash flows of the unleveraged entity be $\mu = 7.5\,\%$ p.a. Following from this, the expected cash flows, assuming normally distributed yields, will be*

$$E\,[\widetilde{CF}] \;=\; V^u\, e^{\mu+\sigma^2/2} \;=\; 109.97\,. \tag{13}$$

The WACC (4) will then be WACC $= 9.97\,\%$ and they are independent of the debt-equity-ratio of the entity. The expected cash flows to the equity holders will be [8]

$$\begin{aligned} E\left[\max(\widetilde{CF} - Z - T, 0)\right] \;&=\; V^u\, e^{\mu+\sigma^2/2}\, N\left(\tfrac{\ln(V^u/(Z+T))+\mu+\sigma^2}{\sigma}\right) \\ &\quad -(Z+T)\, N\left(\tfrac{\ln(V^u/(Z+T))+\mu}{\sigma}\right) \;=\; 30.41 \end{aligned} \tag{14}$$

and the expected cash flows of the creditors may be determined as being

$$E\left[\min(Z + T, \widetilde{CF})\right] \;=\; E\,[\widetilde{CF}] - E\left[\max(\widetilde{CF} - Z - T, 0)\right] \;=\; 79.56\,. \tag{15}$$

Using the market values for equity and debt determined in exhibit (1) and based on definitions (3) and (4) we may calculate the cost of equity as $k_E^l = 24.09\,\%$ and the cost of debt as $k_D = 5.38\,\%$. According to (12) these costs of debt (borrowing costs) are equivalent to a nominal borrowing rate of $i = 5.97\,\%$.

[7] The only exception known to the authors is the work of *Cooper and Davydenko* (2001).

[8] See, e.g. *Cox and Rubinstein* (1985) p. 323 f.

6 An Entity's Incremental Borrowing Rate

According to IAS 36.A17, the incremental borrowing rate may be used as a starting point to determine the cost of capital. The wording ,,Incremental [sic!] Borrowing Rate" would be redundant if it was not understood as the cost of additional, risky debt. However, if debt is risky, there is a distinction between the nominal borrowing rate and the cost of debt, see equation (12). Unfortunately, appendix A.17 does not clearly state whether a nominal borrowing rate or the incremental cost of debt is meant in this context. In addition, the standard is open to interpretation as to whether this incremental borrowing rate is related to subordinated debt (junior debt) or senior debt (pari passu). However, this question is of vital importance: Incremental (additional) risky debt would require the contracts concerning the old debt to be re-assessed and re-negotiated: The nominal borrowing rate would have to be adjusted according to the new (higher) risk. However, in praxi, this would not normally be the case. It is only possible to satisfactorily interpret appendix A.17 (and thus, IAS 36) when the interrelations between the WACC, the cost of debt and the nominal borrowing rate for subordinated debt are taken into consideration.

To determine the incremental nominal borrowing rate and the incremental cost of debt, we assume that amortization and interest ($Z + T$) will increase by one currency unit if the entity chooses to raise additional debt from another creditor. The claims of the equity holders will decrease by one currency unit accordingly (assuming fixed investing activities). With the method derived in chapters 4 and 5, we are now able to determine the market value of the debt and the related cost of debt as well as the incremental borrowing rate according to this higher debt-equity-ratio. We may determine the nominal borrowing rate for the additional debt by calculating the market value of the debt and the sum of the expected cash flows without the additional debt. The cost of debt for the additional subordinated debt and the related nominal borrowing rates may then easily be determined based on the differences of the two calculations.

Exhibit 3 *Based on the data of exhibit 1 und 2 assume that the contractual amortization increases by one currency unit. After this increase and based on the option pricing equations of chapter 4 the market value of the equity is 23.66 CU and the market value of the debt is 76.34 CU. The expected cash flows are 80.48 CU, determined as in exhibit 2. The cost of debt for all of the debt is 5.43 %, the respective nominal borrowing rate is 6.10 %. If the additional debt is subordinated debt, the situation is different: Assuming that the nominal borrowing rate of the old debt stays the same (as in exhibit 2) the cost of the additional debt may be determined as*

$$(80.48378 - 79.55601)/(76.34171 - 75.49376) - 1 = 9.41\,\%, \tag{16}$$

therefore, requiring a contractual agreement on a nominal borrowing rate of $i_n = 17.93\,\%$.

Figure 3 clearly shows, based on the data of exhibits 1-3, that the nominal borrowing rate is substantially different from the cost of debt k_D at a debt-equity-ratio of $D/E^l = 75.49/24.51 \approx 3$, even if there is only one homogeneous group of creditors. Under these conditions, the nominal borrowing rate would only be an acceptable approximation for the WACC at a debt-equity-ratio as high as $D/E^l \approx 7$. Therefore, the nominal borrowing rate for additional subordinated debt does not seem to be an acceptable approximation for WACC. Even at a debt-equity-ratio as low as $D/E^l \approx 3$, the nominal interest borrowing rate for additional debt would be $i_n \approx 18\,\%$, see exhibit 3. The difference between the nominal borrowing rate for additional debt and WACC will be even larger for higher debt-equity-ratios.

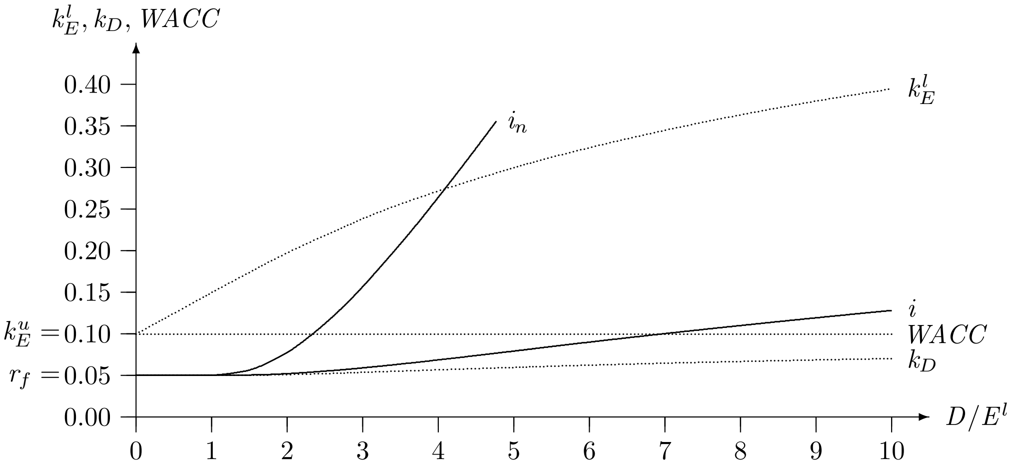

Figure 3: Cost of Capital and Nominal Interest Rate

7 Summary and Conclusions

Determining a value in use in accordance with IAS 36 requires estimating the cost of capital. The guidance in IAS 36.A17 recommends using the weighted cost of capital (WACC), estimated e. g. based on the CAPM. This recommendation is in accordance with the ,,state of the art“ of finance theory. Alternatively, appendix A.17 allows estimating the cost of capital based on the ,,incremental borrowing rate“. This wording is open to interpretation as to whether a nominal borrowing rate for senior or junior debt is meant, i. e. whether additional debt is subordinated or is equally ranked with the old debt. Depending on the debt-equity-ratio, the cost of debt, the nominal borrowing rate and the WACC may differ substantially. The interrelation between these three figures is complex, even under the simplest assumptions. This is particularly true if additional debt is subordinated. Without the information to determine WACC, it is impossible to calculate WACC based on either the cost of debt or the nominal borrowing rate. If the information is known, it would be simpler to directly determine WACC in the first place. IAS 36's recommendation to estimate the cost of capital (as the discount rate to determine the value in use) based on the ,,Incremental Borrowing Rate" is therefore redundant and, moreover, potentially misleading. To avoid substantial estimation errors resulting from IAS 36's guidance, it should be noted that the value in use may only be determined based on WACC and by no means on the ,,Incremental Borrowing Rate". From the point of view of the reporting entity, the guidance in IAS 36.A17 allows earnings management: If a highly leveraged entity is interested in a high impairment, it may determine the value in use using an estimated cost of capital based on the ,,Incremental Borrowing Rate". If the entity is not highly leveraged, it may use WACC as the discount rate instead. If the reporting entity is interested in a low impairment, it may act vice versa, i. e. use WACC if the entity is highly leveraged, and the incremental borrowing rate if it is not.[9]

[9] IAS 8.13 requires the entity to apply its accounting policies consistently. However, this requirement only applies to similar transactions: The entity is not allowed to change the method of determination of the discount rate for similar assets every year, but it may use different methods for different assets in one reporting period, because it does not consider the assets as being similar.

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