VII. Behaviour under Risk and Uncertainty

VII. 1. Assumptions of economic theory

Risk means not to know the true (current or future) „state of the world“ but to know the probabilities. For example:

state: rain, no rain
prob: \( \alpha \), \( 1 - \alpha \)

Possible decisions: Take an umbrella or not

Uncertainty: You do not even know the probabilities.

Solution: Subjective probabilities

Question: - Do individuals always have subjective probabilities?
- Is there such a thing as an objective probability?

Decision principle in economics: Maximize your expected utility!

Example: \[ U(\text{Umbrella, Rain}) = 1, \quad U(\text{Umbrella, no Rain}) = 0 \]
\[ U(\text{no Umbrella, Rain}) = -1, \quad U(\text{no Umbrella, no Rain}) = 2 \]
\[ \Rightarrow \quad EU(\text{Umbrella, -}) = \alpha \cdot 1 + (1 - \alpha) \cdot 0 = \alpha \]
\[ EU(\text{no Umbrella, -}) = \alpha \cdot (-1) + (1 - \alpha) \cdot 2 = 2 - 3\alpha \]

Definition Lottery: \((p, x; 1 - p, y)\) means: \( x \) with prob \( p \), \( y \) with prob \( 1 - p \)

Often: \( x, y = \) amounts of money

Alternatively: \( x, y = \) bundles of goods or states of the world

One can „compose“ lotteries by allowing \( x \) and \( y \) also to be lotteries.

Assumption about the equivalence \((\sim)\) of lotteries.

\[(L1) \quad (1, x; 0, y) \sim x \]
\[(L2) \quad (p, x; 1 - p, y) \sim (1 - p, y; p, x) \]
\[(L3) \quad [q, (p, x; 1 - p, y) ; 1 - q, y] \sim (qp, x; 1 - qp, y) \]

with prob = \( qp \) we get \( x \), with \( q - qp \) we get \( y \).

„Composed lotteries“ allow to deal with more than two states of the world.

The Neumann/Morgenstern/Savage-Axioms (see Varian, Microeconomic Theory, Ch. 11, or another advanced Microeconomics text books) for decisions under risk provide us with assumptions about behaviour which guarantee the existence of a utility function (which can be used for the maximization of expected utility). Such a utility function is defined only up to a linear transformation which changes the origin of the utility function and the unit of utility.
VII. 2. Risk aversion

Example: \( L_1 = 0 \) (with certainty)
\( L_2 = \left( \frac{1}{2} \cdot 5, \frac{1}{2} \cdot -5 \right) \)

Expected values
\( \text{EV}(L_1) = 0 \)
\( \text{EV}(L_2) = \frac{1}{2} \cdot 5 + \frac{1}{2} \cdot (-5) = 0 \)

Expected utilities
\( \text{EU}(L_1) = U(0) = 0 \) (Normalization)
\( \text{EU}(L_2) = \frac{1}{2} U(5) + \frac{1}{2} U(-5) > U(0)? \)

Risk aversion
- Risk neutrality
- Risk preference

Risk aversion
\( L_1 > L_2 \)
\( U(0) > \frac{1}{2} U(5) + \frac{1}{2} U(-5) \)
\( \Rightarrow \) smaller variance is preferred

Risk neutrality
\( L_1 \sim L_2 \)
\( U(0) = \ldots \)
\( \Rightarrow \) utility value = expected utility = larger variance is preferred

Risk preference
\( L_1 < L_2 \)
\( U(0) < \ldots \)

Is there a measure for risk aversion?

The difference between the utility of the expectation value and expected utility depends on the „curvature“ of \( U(x) \). We do not want the measure to depend on linear transformations of \( x \) (see above).

VII. 3. Arrow-Pratt-measure

\[ r(U) = -\frac{U''(x)}{U'(x)} \]
\( x = \text{income or wealth} \)
\( U \rightarrow \tilde{U} = aU + b \)
\[ r(\tilde{U}) = -\frac{\tilde{U}''}{\tilde{U}'} = -\frac{aU''}{aU} = -\frac{U''}{U} \]

So \( r(U) \) is independent of the absolute magnitude of utility and of the „unit of utility“.

Example:
\[ U(x) = x^\alpha \]
\[ \Rightarrow r(U) = -\frac{\alpha(\alpha - 1)x^{\alpha - 2}}{\alpha x^{\alpha - 1}} \]
\[ = -(\alpha - 1) \frac{1}{x} \]
\[ < 0 \text{ for } \alpha > 1 \]
\[ > 0 \text{ for } \alpha < 1 \]
\[ = 0 \text{ for } \alpha = 1 \]
\( \alpha > 1: \) Risik preference  \( \alpha < 1: \) Risik aversion

\( r \) is locally defined, therefore it is a measure for local risk aversion.

**VII. 4. Problems with scale:** Must people be risk neutral with respect to small bets? (Rabin, 2000, Econometrica, 1281-1292)

What would be the consequence if someone always, i.e. for every wealth \( x \), rejects the following lottery \( L \) against the alternative “win/lose 0 Euro”?

\[ L = \text{win Euro } 11 \text{ with prob } = 0.5, \text{ lose Euro } 10 \text{ with prob } = 0.5. \]

The rejection implies

\[
(R) \quad 0.5U(x - 10) + 0.5U(x + 11) < U(x)
\]

Under the assumption that the utility function \( U(x) \) is concave, i.e. \( U''(x) < 0 \) for all \( x \), we have \( U'(x-10) > U'(x) > U'(x + 11) \).

Thus,

\[
U(x - 10) > U(x) - 10U'(x - 10) \\
U(x + 11) > U(x) + 11U'(x + 11)
\]

Together with (R) this implies

\[
0.5 U(x) - 0.5 \cdot 10U'(x - 10) + 0.5U(x) + 0.5 \cdot 11 U'(x + 11) < U(x)
\]

or

\[
U'(x + 11) < \frac{10}{11} U'(x - 10)
\]

or, substituting \( x - 10 \) by \( y \),

\[
U'(y + 21) < \frac{10}{11} U'(y).
\]

Applying this relation again and again, we get

\[
U'(y + n \cdot 21) < \left( \frac{10}{11} \right)^n U'(y).
\]

If \( y = \text{Euro } 100000 \), then
By the same reasoning, $U'(98000)$ is about 10000 times larger than $U'(100000)$.

Consequence

Such a person cannot be compensated by any win in a 50–50 bet where she can lose, say, Euro 3000.

This seems to be nonsense! Someone who owns Euro 100000 should be happy to accept a bet where he could receive another Euro 100000 with 50% probability under the risk of losing Euro 3000 with the same probability!

But then such a person should be practically risk neutral in the case of small bets.

Note, that the same implication is involved in utility functions with constant Arrow-Pratt-measures. In a 50 - 50 bet the loss of $\Delta x$ (see Figure) cannot be compensated by any win.

**Figure:** Utility function with constant Arrow-Pratt measure

**VII. 5. Experiments**

1. **Allais – Paradox**

<table>
<thead>
<tr>
<th>Lottery</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1 Million (sure)</td>
</tr>
<tr>
<td>B</td>
<td>(0.10, 5 Mio; 0.89, 1 Mio; 0.01; 0)</td>
</tr>
<tr>
<td>C</td>
<td>(0.11, 1 Mio; 0.89, 0)</td>
</tr>
<tr>
<td>D</td>
<td>(0.10, 5 Mio; 0.90, 0)</td>
</tr>
</tbody>
</table>

How would you choose?
1. $A > B$, 2. $D > C$?

The first preference means

$$U(1 \text{ Mio}) > 0.1U(5 \text{ Mio}) + 0.89U(1 \text{ Mio}) + 0.01 \cdot U(0)$$

$$\Downarrow$$

$$0.11U(1 \text{ Mio}) > 0.1U(5 \text{ Mio}) + 0.01 \cdot U(0) + 0.89U(0)$$

$$\Downarrow$$

$$C > D$$

Contradiction to your second preference!

2. **Ellsberg paradox**

Please inform yourself via internet or via a textbook.

**VII.6. Prospect theory**


The important elements of Prospect Theory are

(i) a **reference point** from which gains and losses are determined,

(ii) a **value function** over gains and losses,

(iii) a **transformation of objective probabilities**.

(i): The eyes adapt to brightness, the body to temperature, etc. We adapt to comfortable or uncomfortable situations – and try to "make the best out of a situation".

(ii): The value function is S-shaped, i.e. risk averse for gains, risk seeking for losses. At the reference point, the function is steeper for losses than for gains.

**Figure**: The value function of Prospect Theory.

(iii): Low probabilities are over-weighted and high probabilities are under-weighted.
Applications:

1. Endowment effect

In addition, we always have to transform a given situation into the language of Prospect Theory, i.e. we have to describe the relevant decisions (often alternatives) and the reference point.

After we have determined the value of income $v(x)$ and the transformed probabilities $w(p)$, we assume decisions are made according to the expected value (as in the normative theory of decisions under risk):

Choose that “Lottery” $L = (p, x; 1 - p, y)$ with the maximal

$w(p) v(x) + w(1 - p) v(y)$. 

Figure: Empirical decision weights derived from two early studies. 
Source: Handbook of Experimental Economics.
3. Allais-Paradox

Please try to “explain” the Allais-Paradox by means of Prospect Theory.

4. Playing Roulette

What does Prospect Theory predict, when you are in a casino where you can play a series of Roulette games. When will you stop playing? When will you proceed?
5. Pros and cons

Pro:
- Prospect Theory can explain a number of phenomena, often contradicting standard economic theory

Cons:
- There is some arbitrariness in “explaining” the situation, in particular with respect to the reference point
- It is too simple: real evaluations are (sometimes) more complex