Uniform Pricing and the Core in a Public Goods Economy

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Abstract: As an alternative to the Lindahl equilibrium for public goods Hines (2000) has proposed another version of the benefit principle in which distribution of the agent's utility is determined as if each agent had to pay the same public-good price. In this note it is shown by a simple Cobb-Douglas example that Hines' approach may lead to allocations that are not in the core. It is also possible that in a Hines solution poor agents have to make income transfers to some rich agent.

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1. Introduction

In a public-goods economy the most prominent method for making a selection among Pareto optimal allocations is provided by the venerable Lindahl approach in which the agents are confronted with personalized prices for the public good These Lindahl prices are determined such that the agent would unanimously choose the same level of the public good. Even though, on the one hand, the Lindahl solution mimics the price mechanism for private goods, it is, on the other hand, quite different from a market solution since Lindahl prices for a public good normally will not be identical among the agents. In order to overcome this asymmetry Hines (2000) has devised a new method for making a selection among Pareto-efficient public good allocations that is, in a Musgravian style, based on a separation between the decisions on distribution and on allocation. So in Hines' approach that represents a special version of the benefit principle (for a further interpretation see e.g. Kaplow, 2006) distribution of utility is determined by treating every agent *as if* he had to pay the same public-good price. Then that feasible public-good allocation actually is implemented as the *Hines solution* (abbreviated as HS) which entails this distribution of utility.

Having distribution of utility according to a uniform public-good price brings, in some sense, the choice of a public-good allocation closer to that in the case of private goods. But this comes at a price since at the same time HS may be less stable than a Lindahl equilibrium. So we show in this note that − unlike the Lindahl solution − HS may lie outside the core of the economy. Under specific circumstances, it may also happen that some agents make negative contributions to the public good in HS and thus have a double advantage in that type of solution. In Section 2 we describe these effects using a simple numerical example based on Cobb-Douglas preferences. In Section 3 we then try to provide some intuitive explanation for these features of the Hines approach.

2. The Example

Consider a class of public-good economies that are constructed as follows: In each of these economies there is a fixed amount of total income $\overline{Y}$ which, for the sake of simplicity, is normalised to 1. We assume that some specific agent 0 has income Y_0 $(< \overline{Y})$ and that the residual income $Y_s = \overline{Y} - Y_0$ is distributed evenly among an arbitrary number n of other agents $i = 1,...,n.$

As in Hines' (2000) illustrative numerical example every agent $i = 0,1,...,n$ has the same Cobb-Douglas utility function which here will be of the special type $u(y_i, G) = y_i^{1/2} G^{1/2}$ where

y_i denotes agent i's private consumption and G is public-good supply. The public good is produced in the standard way by a constant returns to scale summation technology for which the marginal rate of transformation between the private and the public good is normalised to 1 (see e.g. Bergstrom, Blume and Varian, 1986, or Cornes and Sandler, 1996). Independent of the distribution of total income $\bar{Y} = 1$ and thus, in particular, independent of the size of n, public-good supply in any Pareto-efficient allocation is $G^* = 1/2$.

The constant $\mu(n) = \mu$ that appears in eq. (6) in Hines (2000, p.489) for such an economy is

(1) $$\mu(n) = \frac{1}{2} \cdot \frac{1}{Y_0^2 + n\left(\frac{Y_s}{n}\right)^2} = \frac{1}{2} \cdot \frac{n}{n + z^2}\left(\frac{1}{Y_0}\right)^2$$

where $z := Y_s / Y_0$. Again according to eq. (6) of Hines (2000), private consumption of any agent $i = 1,...,n$ in HS then is

(2) $$y_i^H(n) = \mu(n) \cdot \left(\frac{Y_s}{n}\right)^2 = \frac{1}{2} \cdot \frac{z^2}{n(n + z^2)}$$

Utility of each agent $i = 1,...,n$ in HS thus becomes

(3) $$u_i^H(n) = (y_i^H(n) \cdot G^*)^{1/2} = \frac{1}{2} \cdot \left(\frac{z^2}{n(n + z^2)}\right)^{1/2}$$

which is clearly decreasing in n and increasing in z.

In order to infer the core properties of HS we now compare utility (3) with the utility level that each agent $i = 1,...,n$ would attain in the symmetric efficient standalone solution of this group of agents which is

(4) $$\tilde{u}_i^L(n) = \left(\frac{Y_s}{2} \cdot \frac{Y_s}{2n}\right)^{1/2} = \frac{z}{2(1+z)}\left(\frac{1}{n}\right)^{1/2}$$

since $Y_s = \frac{z}{1+z}$. A short calculation shows that $\tilde{u}_i^L(n) > u_i^H(n)$ as soon as

(5) $$n > 1 + 2z\,.$$

If this condition is fulfilled, which is more likely if *n* is large and *z* is small, the coalition could “block” HS such that HS is not in the core of the economy. If, for example, $z = \frac{1}{3}$, eq. (4) implies that a group consisting of only two agents $i = 1, 2$ is sufficient for having HS outside the core. Note, however, that a single agent ($n = 1$) can never increase his utility by leaving HS.

The reverse of this potential improvement for agents $i = 1, ..., n$ is that a “big” agent 0 may benefit strongly from HS. Private consumption of agent 0 in HS is

(6) $$y_0^H = \mu(n) \cdot Y_0^2 = \frac{1}{2} \cdot \frac{n}{n + z^2}.$$

And his utility is

(7) $$u_0^H(n) = (y_0^H(n) \cdot G^*)^{1/2} = \frac{1}{2} \cdot \left(\frac{n}{n + z^2} \right)^{1/2}$$

which − unlike $u_i^H(n)$ − is increasing in *n* and decreasing in *z*. Since

(8) $$\lim_{n\to\infty} y_0^H(n) = \lim_{n\to\infty} u_0^H(n) = \frac{1}{2}$$

Agent 0’s position in HS converges to that in the extreme allocation in which agent 0 had the total aggregate income $\overline{Y} = 1$ as his private income. This allocation gives agent 0 the maximum utility that can be attained by him if aggregate income $\overline{Y} = 1$ is available. This can be interpreted as a complete redistribution of income to agent 0 if n goes to infinity.

But also with a finite number of *n* agent 0’s private consumption in HS may exceed his initial income Y_0 such that agents $i = 1, ..., n$ not only provide the whole amount of the public good by themselves but also make a positive income transfer to agent 0. It is the result of an easy calculation that $y_0^H(n) > Y_0$ if

(9) $$n > \frac{2z^2}{z-1}.$$

Condition (9) is stronger than condition (5). It needs at least $n = 9$ agents to induce a negative public-good contribution of agent 0. This is obtained when z is in the interval $(\frac{3}{2}, 3)$. If z, however, is outside this interval, a much higher *n* may be required. If $z \leq 1$, i.e. if agent 0 holds at least the same income as the other agents together, it is never possible in our example that agent 0 makes a negative contribution to the public good.

3. An Explanation

The reason why HS may not be in the core and what thus is behind the example given in the previous section can be made more intuitive in the following way: Let total income $\overline{Y}$ be fixed, as before. Now we, however, start with the n agents $i = 1,...,n$ that equally share some part of this income called Y_s and, in the original state, choose the symmetric efficient allocation with the personalized public-good price $1/n$ for each of these agents. Now agent 0 with individual income $Y_0 = \overline{Y} - Y_s$ enters the scene, and we hypothetically assume that also agent 0 had to pay the public-good price $1/n$. The larger *n*, i.e. the smaller this public-good price, the higher agent 0's utility as a price-taker would become. For a very large *n* agent 0 eventually attains a utility level that would not be technically feasible given $\overline{Y}$, such that the corresponding indifference curve lies above the budget line $y_0 + G = \overline{Y}$ in a y_0-*G*-diagram. With indifference curves that are tangential to the coordinate axis such an excessive utility of agent 0 may be brought about independently of agent 0's income share, but the required number of other agents $i = 1,...,n$ is the smaller the higher Y_0 is in relation to Y_s, i.e. − with the notation of our example − the smaller z is.

If agent 0's utility confronted as a price-taker with the public-good price $1/n$ would not be feasible given $\overline{Y}$, the same clearly holds if the common public-good price $\rho^*(n)$ in HS would be lower than $1/n$. To obtain a feasible allocation therefore $\rho^*(n) > 1/n$ is needed which, however, means that agents $i = 1,...,n$ are worse off in HS after agent 0 has joined. Conversely, the coalition consisting of agents $i = 1,...,n$ is able to block HS of the whole economy including agent 0 such that this HS is not in the core. It is more likely that standing alone will be profitable for agents $i = 1,...,n$ if their share in total income is small and is dispersed among a large number *n* of agents. On the other hand, a "big" agent 0 joining a large group of “small” agents then will benefit from HS. These results are reflected in our numerical example but also can be generalized in a straightforward way to a much broader class of utility functions.

If, for some fixed Y_s, the number of agents $i = 1,...,n$ is increased, agent 0’s utility level in HS is growing and converges to some upper limit. It is well possible that this utility level becomes so high that the public-good level that is provided in any feasible and efficient allocation given $\bar{Y}$ is not high enough to satisfy agent 0’s claim. The n compensation for agent 0 has to be achieved by increasing agent 0’s private consumption to a level that lies above his initial income Y_0 which means, as in our numerical example, that it may become possible that agents $i = 1,...,n$ have to make an income transfer in HS. These results may also be generalized though a rather complicated theoretical argument is needed for that.

4. Conclusion

By choosing efficient public-good allocations through the virtual equal pricing scheme as proposed by Hines (2000) the conspicuous difference between Lindahl prices for public goods on the one hand and market prices for private goods on the other can be avoided: Concerning distribution of utility everyone is confronted with the same price in the Hines approach. But this alleged advantage the Hines solution thus has over the Lindahl outcome is accompanied by some adverse effects. So a large group of small agents may benefit from leaving a cooperative arrangement with a big agent in which public-good provision is made according to Hines’ version of the benefit principle. Therefore, it is not ensured that HS − quite unlike the Lindahl equilibrium − is in the core such that the “minimal rationale for everyone to continue to participate” (Foley, 1970. p. 72) may be violated. Moreover, as it has also been shown in this note, it even becomes possible that in HS income transfers from poor to rich agents occur. This stands, however, in serious conflict with the ability-to-pay principle which, besides the benefit principle as stressed by Hines, is another important normative postulate for fair burden sharing in public-good provision.

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