Equal Sacrifice and Fair Burden-Sharing in a Public Good Economy

by

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Abstract: Applying a willingness-to-pay approach known from contingent valuation in environmental economics we first develop an ordinally based specific measure for the size of individual sacrifice that is connected with an agent's contribution to a public good. We then construct a selection mechanism that picks the unique efficient solution among all allocations that have an equal sacrifice as defined in this way. We show that the solution thus obtained not only corresponds to the egalitarian equivalent public good allocation devised by Moulin but also that it has much in common with the much older Lindahl equilibrium. Moreover, the equal sacrifice solutions as characterized in this paper fulfil the ability-to-pay as well as the benefit principle.

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1. Introduction

Since the beginning of the theory of public finance in the 19th century there have been three famous principles of just taxation that are based on different normative ideas (see e.g. Musgrave, 1959, for a historical review). So *equal sacrifice* of taxation demands that taxation leads to the same (absolute or relative) loss of utility for everyone. In this way a symmetrical and thus fair treatment of all citizens is ensured. Taxation according to *ability to pay*, however, requires that personal tax liability should be positively correlated with the taxpayer's income or wealth which gives some kind of vertical equity among people of different financial capacities. In contrast to equal sacrifice as well as to ability-to-pay the *benefit principle* also takes into account the spending of the tax revenue. It postulates that individual tax burden should be related to the utility gain an agent derives from the governmental expenditures that are financed with his taxes. The benefit principle therefore reflects quid pro quo fairness as known from market exchange of private goods.

During the last couple of years these basic concepts for just taxation have attracted more attention in a field that lies outside the framework of taxation theory in its ordinary sense. So it has become a main topic in the political debate and in economic research how to improve the supply of "global public goods", the most prominent of which by now is climate protection (see, e.g., Kaul, Grunberg and Stern, 1999, Kaul et al., 2003, Sandler, 2004, Nordhaus, 2005, Sandmo, 2006, and Kaul and Conceição, 2006). It is a standard result in the theory of public goods that provision of a public good remains inefficiently low when agents (or in the case of an international public good countries) act non-cooperatively (see, e.g., Sandler, 1992, or Cornes and Sandler, 1996, for a detailed explanation of this standard result). In order to overcome this underprovision problem collective action between nations is required which often, as in the case of the Kyoto protocol in climate policy, is regulated by an international convention (especially concerning climate protection see Stern, 2006, pp. 450-467). In particular it has to be stipulated by such an agreement how the contributions to the global public good are to be distributed among the participating countries.

Designing the fundamental structure of such burden-sharing arrangements the venerable principles of just taxation become relevant once again: Countries will only be ready to accept an agreement when their advantage is in line with their financial obligations, i.e. if the benefit principle is fulfilled. Simultaneously, cooperation can only be expected to be successful if no nation feels overburdened as compared to its partners and so a fair distribution of cooperative efforts is achieved (see Sandler, 2004, pp. 77-79). This concern for an equitable treatment of all participants is reflected by the equal sacrifice principle. In particular in the field of climate change policy there is moreover a broad consensus that richer countries have a higher obligation to finance greenhouse gas abatement which might be considered as an application of the ability-to-pay principle.

Despite their importance and their rather casual reference in the literature the principles for just taxation interpreted as guidelines for fair burden sharing have not been incorporated systematically into the theory of public goods. It is therefore the purpose of this paper to characterize an approach through which a particular efficient public good allocation is selected that simultaneously fulfills these three principles. Taking the equal sacrifice postulate as the starting point we will proceed as follows: In Section 2 we first describe how individual sacrifice being connected with a certain individual public good contribution can be measured by adopting a willingness-to-pay technique that is familiar in environmental economics. By this approach individual contributions to the public good originally measured in units of the private good are converted into public-goods equivalents such that the public good serves as the numéraire. Thus, in contrast to the classical equal sacrifice approach in the theory of taxation, a cardinal measure for individual utility is not required. In Section 3 we first establish some basic properties of this sacrifice measure which are used throughout the paper. In Section 4 then the equity norm is applied to determine the set of public-good allocations for which the level of this sacrifice is identical among all agents. Imposing allocative efficiency for the public good allocation, i.e. the Samuelson rule, as a further normative postulate then gives the desired choice mechanism for public-good allocations. In Section 5 it is first demonstrated that this mechanism corresponds to Moulin´s egalitarian-equivalent solution concept (see Moulin, 1987, 1995) such that an alternative justification for this selection mechanism is provided which is more closely related to standard ideas of equal treatment and to he standard concepts of Public Finance. In this way it also becomes possible to draw a parallel between the egalitarian equivalent solution in a public goods economy and the classical Lindahl equilibrium which is also done in Section 5. In Section 6 we finally show that the equal sacrifice selection rule described in this paper also satisfies the benefit principle and the ability-to-pay criterion such that it indeed incorporates the three fundamental principles for fair burden sharing. (An empirical account of burden sharing in international environmental agreements is given by Lange, Vogt and Ziegler, 2007.)

2. Measuring Individual Sacrifice of Public Good Contribution

We consider a standard public-good economy consisting of n agents $i=1,...,n$ (see the classical treatments in Bergstrom, Blume and Varian, 1986, and Cornes and Sandler, 1996). Agent i

is endowed with an amount y_i of the private good, his income. Total income of all agents is denoted by $Y = \sum_{i=1}^{n} y_i$. The utility function of agent *i* is $u_i(x_i, G)$ where x_i is agent i's level of private consumption and G is public-good supply. Each utility function is at least twice continuously differentiable and strictly monotone increasing in both variables, it is strictly quasi-concave and both the private and the public good are assumed to be non-inferior. To avoid corner solutions we furthermore suppose that in a x_i-G-diagram the indifference curves of all agents are tangential to the coordinate axis as e.g. in the Cobb-Douglas case. Given the utility function $u_i(x_i, G)$ the marginal rate of substitution between the public and the private good at some point (x_i, G) is denoted by $\pi_i(x_i, G) = \frac{\partial u_i / \partial G}{\partial u_i / \partial x_i}$.

The public good is produced by a constant returns to scale summation technology: If agent i contributes $g_i := y_i - x_i$ to the public good the total supply of the public good is given by

$$(1) \qquad G = \sum_{i=1}^{n} g_i .$$

Among all allocations that fulfil the budget constraint (1) we want to characterize those in which there is an equal sacrifice for each agent. Applying this normative concept first of all requires that the size of personal sacrifice is measured in an adequate way. In this context the simplest approach would be to identify agent i's sacrifice with the absolute level of his contribution g_i. But such a specification of sacrifice is only compatible with ethical intuition when all agents are completely identical, i.e. have the same income and the same preferences. Otherwise, one would expect that a smaller income or a lower preference for the public good should increase agent *i*'s subjective burden connected with some given contribution level g_i since this contribution then is harder to bear for her.

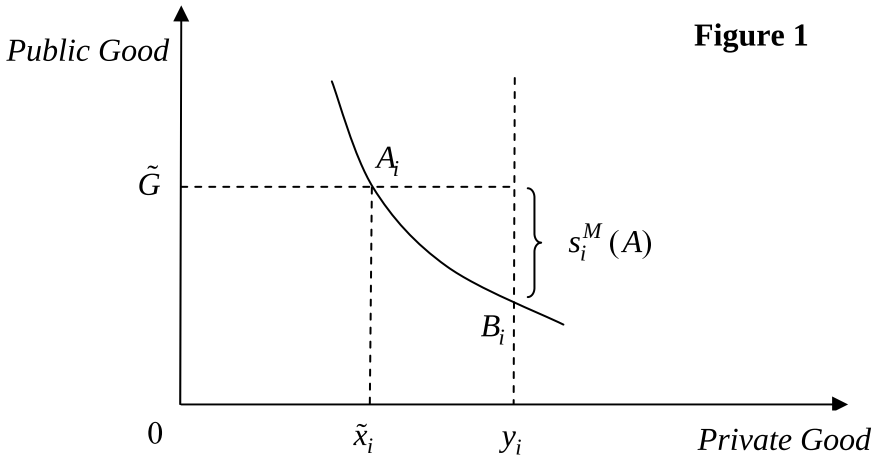
The problem of finding an adequate measure of subjective individual sacrifice already appeared in the classical treatment of equal sacrifice of taxation where sacrifice was related to the loss of utility of income and not to income itself such that that utility has to be cardinally measurable. (See for a modern treatment of the classical equal sacrifice approach Mitra and Ok, 1996, or Moyes, 2003). In the present paper, in which utility of the agents also depends on public good consumption, a measure of agent $i's$ personal sacrifice is obtained by constructing a public-good equivalent to his contribution g_i. As we take the public good as a numéraire the problem of having to make use of cardinal measurability of utility that has been pertinent in the classical equal sacrifice approaches is avoided.[1]

Definition 1: Let $A = (\tilde{x}_1, ..., \tilde{x}_n, \tilde{G})$ be some allocation that is feasible according to (1). The individual sacrifice $s_i^M(A)$ that agent *i* makes in the allocation A is determined by

(2) $$u_i(y_i, \tilde{G} - s_i^M(A)) = u_i(\tilde{x}_i, \tilde{G}).$$

The meaning of Definition 1 is visualised in Figure 1.

Figure 1



By Definition 1 individual public-good contributions g_i are converted into equivalent public-good units and thus made comparable. This method for measuring the personal sacrifice is analogous to the assessment of individual willingness to pay well-known from contingent valuation studies in environmental economics (see e.g. Ebert, 1993, or Kolstad, 2000, pp 291-294). So, agent i's sacrifice $s_i^M(A)$ in a given allocation $A = (\tilde{x}_1, ...\tilde{x}_n, \tilde{G})$ is elicited as the answer to a willingness-to-pay question by which agent *i* is asked how much of the public good she would be ready to give up if − starting from her position $(\tilde{x}_i, \tilde{G})$ − he could simultaneously reduce his public-good contribution to zero. Then agent *i* becomes indifferent between his position $A_i = (\tilde{x}_i, \tilde{G})$ attained in *A* and the position $B_i = (y_i, \tilde{G} - s_i^M(A))$ where private consumption is identical with the initially given income y_i and public-good supply re-

[1] See Neill (2000) for an alternative approach for measuring individual sacrifice in a public-goods economy that, as in the conventional treatments, refers to differences in cardinal utility.

duced by the sacrifice level $s_i^M(A)$. In an alternative interpretation $s_i^M(A)$ indicates agent i's willingness to pay (in units of the public good) for an increase of private consumption from $\tilde{x}_i$ to y_i.

Let – for any utility level $\bar{u}_i$ of agent i – $\varphi_i^h(x_i,\bar{u}_i)$ denote the inverse Hicksian demand function for the private good which coincides with the marginal rate of substitution between the private and the public good $1/\pi_i(x_i,G)$ when (x_i,G) varies along the given indifference curve $\bar{u}_i$. Then $s_i^M(A)$ can be represented as an area below the inverse Hicksian demand function, i.e. as

(3) $$s_i^M(A) = \int_{x_i}^{y_i} \varphi_i^h(x_i, u_i(\tilde{x}_i,\tilde{G}))dx_i = \tilde{g}_i \int_{x_i}^{y_i} \frac{\varphi_i^h(x_i,u_i(\tilde{x}_i,\tilde{G}))}{\tilde{g}_i} dx_i ,$$

where $\tilde{g}_i := y_i - \tilde{x}_i$ is agent i's public-good contribution in the allocation *A*. Thus the sacrifice of agent i in allocation *A* is obtained as this agent's contribution to the public good weighted by the *average* marginal rate of substitution measured along the indifference curve $u_i(\tilde{x}_i,\tilde{G})$.

3. Properties of the Sacrifice Measure

In this section we want to show how the level of the sacrifice depends on his income, his preferences as well as on the position $(\tilde{x}_i,\tilde{G}) = (y_i - \tilde{g}_i,\tilde{G})$ that some agent i has in a certain allocation *A* as well as on his income y_i and his preferences $u_i(x_i,G)$. In the following four steps of the analysis, we will vary only one of the parameters $\tilde{g}_i, \tilde{G}$ and y_i or the utility function $u_i(x_i,G)$ while keeping all other three constant. Part of the adjustment that is required by the transition from the original allocation *A* to a new feasible allocation called A' then have to be made by the other agents. The change of agent i's sacrifice, however, is not affected by the precise nature of the adjustments of the other agents such that they have not to be described explicitly:

(i) If agent's public good contribution is increased from $\tilde{g}_i$ to $\tilde{g}_i'$ his sacrifice obviously increases since the new indifference curve $u_i(\tilde{x}_i',\tilde{G})$ is lower than the original indifference curve $u_i(\tilde{x}_i,\tilde{G})$.

(ii) If public-good supply grows from $\tilde{G}$ to $\tilde{G}'$ the argument is a little more complicated and crucially depends on the normality assumption. Letting $\tilde{u}_i := u_i(\tilde{x}_i, \tilde{G})$ and $\tilde{u}_i' := u_i(\tilde{x}_i, \tilde{G}')$ we consider the two inverse Hicksian demand functions $\varphi_i^h(x_i, \tilde{u}_i)$ and $\varphi_i^h(x_i, \tilde{u}_i')$ that correspond to these utility levels, respectively. From $\tilde{G}' > \tilde{G}$, we have $\tilde{u}_i' > \tilde{u}_i$, Thus normality straightforwardly implies that $\varphi_i^h(x_i, \tilde{u}_i') \geq \varphi_i^h(x_i, \tilde{u}_i)$ holds for all x_i, such that the indifference curve through $(\tilde{x}_i, \tilde{G}')$ is everywhere steeper than that through $(\tilde{x}_i, \tilde{G})$ (see Appendix A1 for details). Using our representation formula (3), this allows for a comparison of the new sacrifice $s_i^M(A')$ (with the new position $(\tilde{x}_i, \tilde{G}')$ of agent i) and the original sacrifice level $s_i^M(A)$.

(4) $$s_i^M(A') = \int_{\tilde{x}_i}^{y_i} \varphi_i^h(x_i, \tilde{u}_i')dx_i \geq \int_{\tilde{x}_i}^{y_i} \varphi_i^h(x_i, \tilde{u}_i)dx_i = s_i^M(A)$$

(iii) If income of agent i is increased from y_i to y_i' the effect on the sacrifice level again rests upon normality. Letting now $\tilde{u}_i' := u_i(y_i' - \tilde{g}_i, \tilde{G})$ we consider the inverse Hicksian demand function $\varphi_i^h(x_i, \tilde{u}_i')$. If a horizontal translation by $t := y_i' - y_i$ is made, normality implies $\varphi_i^h(x_i, \tilde{u}_i) \leq \varphi_i^h(x_i - t, \tilde{u}_i)$, i.e. moving to the right makes indifference curves flatter (see again Appendix A1). Denoting $\tilde{x}_i' = y_i' - \tilde{g}_i$ we then get the following estimate:

(5) $$s_i^M(A') = \int_{\tilde{x}_i'}^{y_i'} \varphi_i^h(x_i, \tilde{u}_i')dx_i \leq \int_{\tilde{x}_i'}^{y_i'} \varphi_i^h(x_i - t, \tilde{u}_i)dx_i = \int_{\tilde{x}_i}^{y_i} \varphi_i^h(x_i, \tilde{u}_i)dx_i = s_i^M(A).$$

This means that agent *i*'s sacrifice becomes smaller if his income is increased.

(iv) Finally, we suppose that agent i is substituted by another type of agent with a utility function $u_i'(x_i, G)$ which represents a weaker preference for the public good than the original utility function $u_i(x_i, G)$. This intensification of preferences for the public good is described by the assumption that the new utility function everywhere exhibits a higher marginal willingness to pay for the public good, i.e. that

(6) $$\frac{\partial u_i' / \partial G}{\partial u_i' / \partial x_i} > \frac{\partial u_i / \partial G}{\partial u_i / \partial x_i}$$

holds for all consumption bundles (x_i, G). This condition means that the indifference curve $\tilde{u}_i' := u_i'(\tilde{x}_i, \tilde{G})$ passing through the point $(\tilde{x}_i, \tilde{G})$ is flatter than the original indifference curve $\tilde{u}_i = u_i(\tilde{x}_i, \tilde{G})$. The two indifference curves $\tilde{u}_i$ and $\tilde{u}_i'$ cannot cross twice because this would violate assumption (6). So, the indifference curve $\tilde{u}_i'$ must lie above the indifference curve $\tilde{u}_i$ right to $\tilde{x}_i$ which clearly implies that agent i´s sacrifice is reduced.

We summarize these findings as follows:

Proposition 1: The individual sacrifice of an agent becomes higher if

(i) the public-good contribution, or

(ii) total public-good supply increases.

The individual sacrifice of an agent is lower if

(iii) his income is higher, or

(iv) his preferences for the public good become stronger.

4. Equal Sacrifice Allocations

Having developed a concept for the measurement of sacrifice it is now straightforward to characterize *equal* sacrifice allocations. This is made precise by the next definition.

Definition 2: Let an income distribution $(y_1, ..., y_n)$ and preferences $(u_1, ..., u_n)$ be given. A feasible allocation $A = (\tilde{x}_1, ..., \tilde{x}_n, \tilde{G})$ is called an *equal sacrifice allocation* when there is some sacrifice level $s > 0$ such that

(7) $$s_i^M(A) = s \qquad \text{for all } i = 1, ..., n.$$

In order to show that such equal sacrifice solutions exist we use the following construction in which we start with some public good level $G \in]0, Y]$. We then define for any sacrifice level $s \in [0, G[$ a private consumption level $\breve{x}_i(s, G)$ of agent i by letting

(8) $$u_i(\breve{x}_i(s,G),G) = u_i(y_i, G-s)\,.$$

Thus, as depicted in Figure 2, $\breve{x}_i(s,G)$ is agent i´s private consumption level when public-good supply is G and this agent should bear the given sacrifice s.

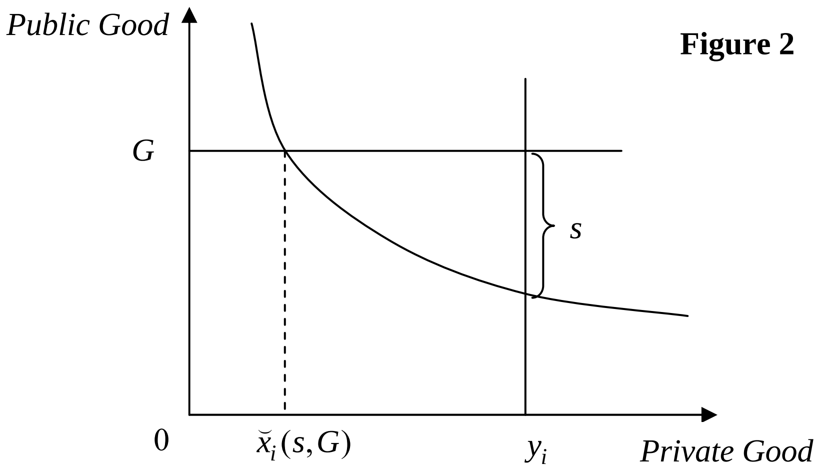


Figure 2

Given our assumption that all indifference curves are tangential to the G-axis a unique private-good consumption level $\breve{x}_i(s,G)$ exists for all $G \in]0,Y]$ and all $s \in [0,G[$. Obviously, $\breve{x}_i(s,G)$ is strictly decreasing in s for a given public-good level G and $\lim_{s \to G} \breve{x}_i(s,G) = 0$ holds for any agent $i = 1,...,n$. Moreover, for a fixed sacrifice level s, $\breve{x}_i(s,G)$ is increasing in G, since it follows from parts (i) and (ii) of Proposition 1 that otherwise the sacrifice level would increase.

Having established these properties of the function $\breve{x}_i(s,G)$ we now consider the function

(9) $$H(s,G) := \sum_{i=1}^{n} \breve{x}_i(s,G) + G\,.$$

The function $H(s,G)$ defined by (8) describes how much aggregate income would be required if public-good supply were G and all agents $i = 1,...,n$ had the equal sacrifice level s. The function $H(s,G)$ is differentiable in both variables and strictly decreasing in s, and from the properties of the functions $\breve{x}_i(s,G)$ it follows that $H(0,G) = Y + G > Y$ and $\lim_{s \to G} H(s,G) = G < Y$. Thus, by continuity and monotonicity of $H(s,G)$ in G the mean-value theorem implies, that there exists a unique value of sacrifice $s^M(G)$ such that

(10) $$H(s^M(G),G) = Y\,.$$

Hence,, there exists a unique equal sacrifice allocation $(\breve{x}_1(s^M(G),G),...,\breve{x}_n(s^M(G),G),G)$ that fulfils the budget constraint (1) with the public good supply G.

By equation (10) the function $s^M(G)$ is implicitly defined for all public good levels in $]0,Y[$, and since $\lim_{G \to 0} s^M(G) = 0$ and $\lim_{G \to Y} s^M(G) = Y$ this function takes on any value in this interval. Furthermore, from totally differentiating equation (9) we obtain

(11) $$\frac{\partial s^M(G)}{\partial G} = -\frac{\partial H / \partial G}{\partial H / \partial s} > 0.$$

This inequality follows from $\partial H / \partial s < 0$ and $\partial H / \partial G > 0$, which holds since each $\breve{x}_i(s,G)$ is increasing in G. As the function $s^M(G)$ thus is strictly increasing it can be inverted. The inverse function of $s^M(G)$, which is called $G^M(s)$, then is defined on $]0,Y[$ and it is strictly increasing, too. This yields the following result.

Proposition 2: For each $s \in]0,Y[$ there is a unique feasible allocation in which all agents have the equal individual sacrifice s.

Proof: Given s let public-good supply be $G^M(s)$ and private consumption of agent i be $x_i^M(s) := \breve{x}_i(s,G^M(s))$. Then, by the construction above, $(x_1^M(s),...,x_n^M(s),G^M(s))$ is a feasible allocation in which all agents have the same sacrifice level s. As $G^M(s)$ is a strictly increasing function the sacrifice level must be different from s in any other feasible equal sacrifice allocation which shows uniqueness.

QED.

5. The Choice Mechanism

Through Proposition 1 it becomes clear that they are infinitely many equal sacrifice solutions that could, depending on the sacrifice level *s*, be described by an "equal-sacrifice curve" $(x_1^M(s),...,x_n^M(s),G^M(s))$ in the $\mathbb{R}^{n+1}$-space (see, e.g., Schlesinger and Sullivan, 1986, for a similar construction in the Kolm-triangle for the two-person case). We now want to show that on this curve there is one single point which gives a Pareto-optimal allocation.

For a proof consider the marginal rates of substitution between the public and the private good along the equal sacrifice curve, i.e. we denote $\pi_i^M(s) := \pi_i(x_i^M(s),G^M(s))$ for each agent $i = 1,\dots,k$ and each sacrifice level $s \in]0,Y[$. From the supposed tangency properties of the indifference curves of all agents we have $\lim_{s\to Y} \pi_i^M(s) = 0$ as $\lim_{s\to Y} x_i^M(s) = 0$ for all agents $i = 1,...,n$. In order to apply the Samuelson rule we now denote $\prod^M(s) := \sum_{i=1}^{n} \pi_i^M(s)$ as the sum of these marginal rates of substitution. Clearly, $\lim_{s\to Y} \prod^M(s) = 0$ and $\lim_{s\to 0} \prod^M(s) = \infty$ holds such that there is some $s^* \in]0,Y[$ for which $\prod^M(s^*) = 1$. The feasible equal sacrifice allocation $(x_1^M(s^*),...,x_n^M(s^*),G^M(s^*))$ then fulfils the Samuelson condition and thus is Pareto-optimal.

In order to show that $(x_1^M(s^*),...,x_n^M(s^*),G^M(s^*))$ is the *unique* efficient allocation in the economy under consideration we need a separate argument. Note that in an equal sacrifice allocation the utility levels of different agents can never move in an opposite direction when the sacrifice level *s* is changed. This is obvious since − according to equations (2) and (7) and the definition of $G^M(s)$ − utility of each agent must change in the same direction as $G^M(s) - s$.

Now suppose that there are two different sacrifice levels s^* and s^{**} for which $\prod^M(s^{**}) = \prod^M(s^*) = 1$ holds such that two Pareto-optimal allocations would exist. It is a direct consequence of our observation concerning the parallel change of all agent's utility that $u_i(x_i^M(s^*),G^M(s^*)) = u_i(x_i^M(s^{**}),G^M(s^{**}))$ holds for all $i = 1,...,n$, i.e. all agents have the same utility in both equal sacrifice solutions. Otherwise, a contradiction to the supposed Pareto optimality of the two equal sacrifice allocations would result.

Without loss of generality, $s^{**} > s^*$ may be assumed such that from strict monotonicity of the function $G^M(s)$ we get $G^M(s^{**}) > G^M(s^*)$. Having the same utility levels in both allocations thus requires $x_i^M(s^{**}) < x_i^M(s^*)$ for all agents $i = 1,...,n$. From the assumed normality of preferences we then get $\pi_i^M(s^{**}) = \pi_i(x_i^M(s^{**}), G^M(s^{**})) < \pi_i(x_i^M(s^*), G^M(s^*)) = \pi_i^M(s^*)$ for all agents $i = 1,...,n.$ This gives $1 = \Pi^M(s^{**}) = \sum_{i=1}^{n} \pi_i^M(s^{**}) < \sum_{i=1}^{n} \pi_i^M(s^*) = \Pi^M(s^*) = 1$ which is a contradiction. So we can conclude:

Proposition 3: There is a unique sacrifice level s^* such that the equal sacrifice allocation $(x_1^M(s^*),...,x_n^M(s^*), G^M(s^*))$ is Pareto optimal.

Using Proposition 3 the mechanism that picks an equal sacrifice solution is now characterized as follows:

Definition 3: Let a public goods economy be given by the income distribution $(y_1,...,y_n)$ and preferences $(u_1,...,u_n)$. Then the *equal sacrifice solution* for this public goods economy is defined as $(\hat{x}_1^M,...,\hat{x}_n^M,\hat{G}^M) = (x_1^M(\hat{s}^M),...,x_n^M(\hat{s}^M), G^M(\hat{s}^M))$ where the sacrifice level $\hat{s}^M := s^*$ is determined according to Proposition 3.

Given normality the equal sacrifice solution as characterized by Definition 3 is well-defined and unique.

5. Comparison to the Literature

It is now straightforward that the equal sacrifice solution according to Definition 3 coincides with the egalitarian- equivalent allocation of the given economy (see Moulin, 1987, 1995). Given an income distribution $(y_1,...,y_n)$ and preferences $(u_1,...,u_n)$ define $\bar{G}^M := \hat{G}^M - \hat{s}^M$. From condition (2) we then have

$$(13) \qquad u_i(\hat{x}_i^M, \hat{G}^M) = u_i(y_i, \bar{G}^M)$$

such that $\bar{G}^M$ is the egalitarian-equivalent public-good supply in the sense of Moulin. Using a different line of the argument than Moulin himself, we have thus been able to link the egalitarian equivalent solu6tion concept to the equal sacrifice principle. (For justifications of the Moulin solution see – besides Moulin, 1987, himself – Maniquet and Sprumont, 2004)

In this way it becomes also possible to compare Moulin´s approach with the much older Lindahl solution concept. To this end we define – quite analogous to (2) – for any given feasible allocation $A=(\tilde{x}_1,...,\tilde{x}_n,\tilde{G})$ an alternative *Lindahl-sacrifice* by letting

$$(14) \qquad s_i^L(A) := \frac{\tilde{g}_i}{\pi_i(\tilde{x}_i,\tilde{G})}.$$

where $\tilde{g}_i = y_i - \tilde{x}_i$ again denotes agent i´s contribution to the public good in *A*.

The sacrifice level according to (14) is measured by using the *marginal* valuation of the private good in units of the public good in the position $(\tilde{x}_i,\tilde{G})$ instead of the total willingness to pay as in the Moulin concept (see, e.g. Ebert, 2003, and Ebert and Tillmann, 2006, for a general discussion of the marginal valuation approach in a public good economy). This Lindalian measurement device is visualized in Figure 3.

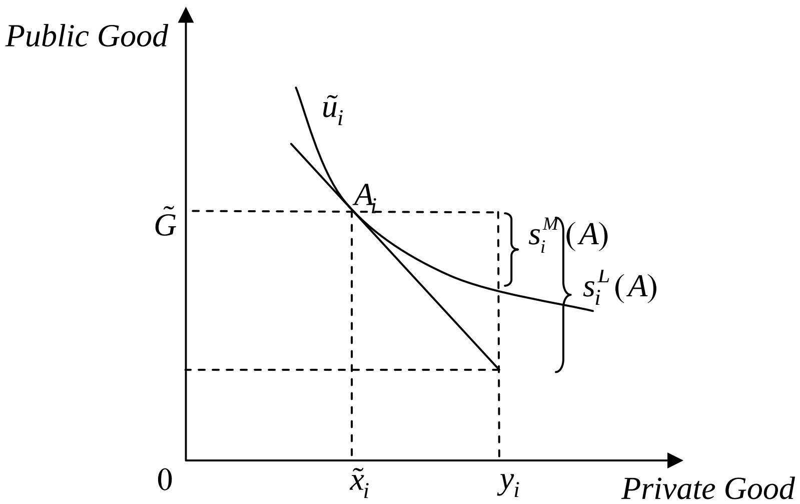


Figure 3

It is obvious from Figure 3 that having strictly convex indifference curves $s_i^L(A) > s_i^M(A)$ is automatically implied, i.e. the level of the Lindahl sacrifice always exceeds the level of the sacrifice.

It again follows from normality that the Lindahl sacrifice has the same properties as stated by Proposition 1. We can also identify equal sacrifice allocations that are based on the Lindahlian sacrifice concept. The corresponding choice mechanism then picks an allocation $(\hat{x}_1^L,...,\hat{x}_n^L,\hat{G})$ that implies an equal Lindahl sacrifice for all agents and is efficient as well. It has been shown elsewhere (see Buchholz and Peters, 2007) that the Lindahlian equal sacrifice allocation $(\hat{x}_1^L,...,\hat{x}_n^L,\hat{G}^L)$ is identical with the standard Lindahl equilibrium that would be chosen if the agents $i = 1,...,n$ acted as price-takers and agent i were confronted with the personalized Lindahl price $\hat{p}_i = \pi_i(\hat{x}_i^L, \hat{G}^L)$.

Even though the sacrifice measures s_i^M and s_i^L are conceptually different, they may yield the same efficient equal sacrifice solutions under specific circumstances. This is e.g. the case if all agents have identical Cobb-Douglas preferences. Then, in both equal sacrifice solutions, the public-good contributions of all agents $i = 1,...,n$ are proportional to there income levels y_i (see the Appendix A2 for a detailed analysis of the Cobb-Douglas case).

6. Properties of the Equal Sacrifice Solutions

In this Section we show that the equal sacrifice solution as characterized in this paper fulfils both the ability to pay principle and the benefit principle. (Concerning the empirical relevance of both principles in the case of global public goods see Barrett, 2006, pp. 365-366.) To make this precise we first of all have to define what these principles are to mean exactly.

Concerning ability to pay we assume that two agents j and k have the same utility function but differ with respect to their income, such that without loss of generality $y_k > y_j$ holds. If some arbitrary choice mechanism E picks an allocation $(\hat{x}_1^E,...,\hat{x}_n^E,\hat{G}^E)$ with individual public-good contributions $\hat{g}_i^E := y_i - \hat{x}_i^E$, this mechanism is said to fulfil the (weak) ability to pay principle if $\hat{g}_k^E \geq \hat{g}_j^E$ holds. The richer agent k makes no lower contribution to the public good than the poorer agent j.

Analogously the weak benefit principle requires that, given the same income level, an agent with a stronger preference for the public good, as defined by reference to marginal willingness to pay in condition (6), should make no smaller contribution to the public good than an agent with a weaker preference (see Hines, 2000, for a general discussion of the benefit principle). If this condition is met and, additionally, $y_j = y_k$ holds the benefit principle is said to be fulfilled for a mechanism E if and only if $\hat{g}_k^E \geq \hat{g}_j^E$.

It is now a straightforward consequence of Proposition 1 that both principles are fulfilled for equal sacrifice solutions: Assume that the public-good contribution of agent *k* would be smaller than that of agent *j* if the income of agent *k* were higher than that of an agent *j* or agent *k*'s preferences for the public good are stronger than that of agent *j*. Combining the results of Proposition 1 (i) with those in Proposition 1 (iii) or (iv), respectively, then implies that agent *k* would have to bear a lower sacrifice than agent *j* which is a contradiction to the equal sacrifice assumption.

7. Conclusion

This paper has shown how in a standard public goods economy the venerable equal sacrifice principle can be applied to make a selection among efficient allocations. Unlike the traditional literature we, however, did not make use of losses in cardinally measurable utility as an indicator of individual sacrifice but instead obtained a sacrifice measure by transforming the individual expenses for the public good into public-good equivalents. The method by which this transformation was made has been borrowed from willingness-to-pay assessment that is well known from contingent valuation techniques in environmental economics. The public-good allocations that show an equal sacrifice as defined in this way and are Pareto optimal as well turn out to be identical with the egalitarian equivalent solutions as conceived by Moulin (1987). Moreover, they fulfil the ability to pay and the benefit principle properly defined.

The novel justification of the egalitarian equivalent solution concept provided in this paper also makes it possible to recognize its similarity with the classical Lindahl equilibrium since also the Lindahl mechanism can be put down to the equal sacrifice principle. The difference, however, is that assessment of the sacrifice as made in our approach is based on *total* willingness to pay or, as an alternative interpretation, *average* valuation of the public good whereas the Lindahl solution rests upon valuation of public-goods contribution according to *marginal* willingness to pay. In special cases both equal sacrifice solution may coincide but generally they will be different.

Measuring individual sacrifice by total valuations as in the present paper takes more information on individual preferences into account than the Lindahl approach in which assessment of sacrifice only is based on marginal willingness to pay in a single point. In a world of full information (that also underlies the standard treatments of the Lindahl and the egalitarian equivalent solution concepts) the equal sacrifice solution as characterized in this paper therefore is based on a more accurate valuation of individual sacrifice than its Lindahl counterpart.

Appendix

A1: Steepness of Indifference Curves

Consider agent i and fix some level x_i of her private consumption. Let, as in the main text, $\overline{u}_i'$ and $\overline{u}_i''$ be two utility levels of agent i with $\overline{u}_i'' > \overline{u}_i'$. By G' and G'' we then denote the levels of public-good supply for which $u_i(x_i, G') = \overline{u}_i'$ and $\overline{u}_i'(x_i, G'') = \overline{u}_i''$ holds. Now assume $p_i'' := \varphi_i^h(x_i, \overline{u}_i'') < \varphi_i^h(x_i, \overline{u}_i') =: p_i'$, i.e. that the indifference curve $\overline{u}_i''$ at (x_i, G'') is flatter than the indifference curve $\overline{u}_i'$ at (x_i, G'). Thus, as depicted in Figure 5, agent i endowed with the income $y_i'' := x_i + p_i'G''$ and confronted with the public-good price p_i' would demand less of the private good than x_i.

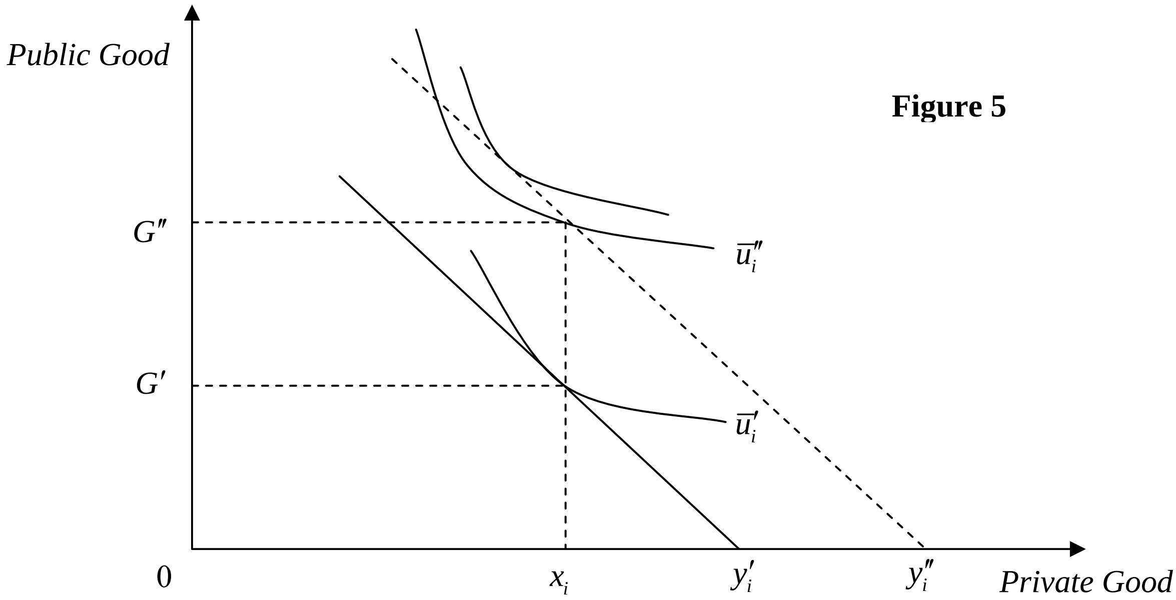


Figure 5

Since x_i is agent i's private good demand given the income $y_i' = x_i + p_i'G'$ and the public good price p_i' and clearly $y_i' < y_i''$ holds this would contradict the assumption that the private good is normal.

Since indifference curves are convex it is a straightforward implication of this argument that indifference curves become steeper if public good consumption grows and private good consumption simultaneously falls.

A2: The Equal Sacrifice Solution in the Cobb-Douglas Case

Let n agents $i = 1, ..., n$ be given by their income levels $y_1, ..., y_n$ and their Cobb-Douglas utility functions $u_i(x_i, G) = x_i G^{\rho_i}$. In the equal sacrifice solution $(\hat{x}_1^M, ..., \hat{x}_n^M, \hat{G}^M)$ the common equal sacrifice level $\hat{s}^M$ must fulfil $\hat{x}_i^M \hat{G}^{\rho_i} = y_i(\hat{G}^M - \hat{s}^M)^{\rho_i}$ for all agents $i = 1,...,n$ which yields $\hat{s}^M = \hat{G}^M(1-(\frac{\hat{x}_i^M}{y_i})^{\frac{1}{\rho_i}})$ for the common sacrifice level. For individual private consumption we obtain $\hat{x}_i^M = A^{\rho_i} y_i$, where $A := \frac{\hat{G}^M - \hat{s}^M}{\hat{G}^M} < 1$ is a constant for the given public-goods economy. The individual public-good contributions then are $\hat{g}_i^M = (1 - A^{\rho_i}) y_i$.

This expression clearly confirms the results of Proposition 4 in the main text for the Cobb-Douglas case: If two agents *j* and *k* have the same preferences, i.e. $\rho_j = \rho_k$ holds, but $y_k > y_j$, then $\hat{g}_k^M > \hat{g}_j^M$ such that the agent with the higher income makes a higher contributions to the public good and ability to pay is fulfilled. If, on the other hand, two agents j and k have the same income $y_j = y_k$ but $\rho_k > \rho_j$, we have again $\hat{g}_k^M > \hat{g}_j^M$, i.e. the agent with the stronger preference for the public good makes a higher contribution which gives the benefit principle.

We now compare our equal sacrifice solution with the Lindahl equilibrium $(\hat{x}_1^L,...,\hat{x}_n^L,\hat{G}^L)$ that results in the same situation. Here, individual public good contributions are $\hat{g}_i^L = \frac{\rho_i}{1+\rho_i} y_i$ for agents $i = 1,...,n$. When all agents have the same preferences, such that $\rho = \rho_i$ for $i = 1,...,n$, it directly follows from our formulas that public-good contributions must be proportional to the individual income level in both solutions. Since efficiency of both outcurves requires $\hat{G}^M = \hat{G}^L = \frac{\rho}{1+\rho} Y$ where $Y = \sum_{i=1}^{n} y_i$ is total income, both equal sacrifice solutions coincide when all agents have the same Cobb-Douglas preferences.

Having agents with different Cobb-Douglas preferences the equal sacrifice solution $(\hat{x}_1^M,...,\hat{x}_n^M,\hat{G}^M)$, however, may differ from $(\hat{x}_1^L,...,\hat{x}_n^L,\hat{G}^L)$. This is demonstrated by the following simple example: Let $n = 2$, $y_1 = y_2 = 1$ and $\rho_1 = 1$ and $\rho_2 = 2$. Then clearly $\hat{x}_1^L = \frac{1}{2}$, $\hat{x}_2^L = \frac{1}{3}$, $\hat{g}_1^L = \frac{1}{2}$, $\hat{g}_2^L = \frac{1}{3}$ and $\hat{G}^L = \frac{7}{6} = 1{,}17$. The utility levels of both agents are $\hat{u}_1^L = 0.58$ and $\hat{u}_1^L = 0.45$. On the other hand, $\hat{x}_2^M = (\hat{x}_1^M)^2$ which – combined with the feasibility condition and the Samuelson rule for efficiency – leads to the quadratic equation $3(x_1^M)^2 + 2\hat{x}_1^M - 2 = 0$. Solving this equation for $\hat{x}_1^M$ yields $\hat{x}_1^M = 0.55$, which then implies

$\hat{x}_2^M = 0.3$, $\hat{g}_1^M = 0.45$, $\hat{g}_2^M = 0.7$ and thus $\hat{G}^M = 1.15$. This shows that both solutions need not be identical. For the utility levels we obtain $\hat{u}_1^M = 0.63$ and $\hat{u}_2^M = 0.4$ such that agent 1 with the lower preference for the public good is worse off in the Lindahl solution where for agent 2 the reverse result holds.

In the general case with non-homogenous Cobb-Douglas preferences an explicit comparison between the two solutions is difficult to make since no closed form expression for the Moulin outcome exists in this case.

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