DIALOG PROGRAM FOR THE TWO-CRITERIAL DYNAMIC LOT SIZE MODEL

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The dynamic lot size model with two objectives is studied. The criterion of minimizing the sum of set-up costs and holding costs is complemented by the minimization of the stock. Solutions which are efficient with respect to these two objectives can be derived from parametric one-criterial models with combined objective functions. A complete set of efficient solutions which are distinct in their objectives can be found by a BASIC dialog program for personal computers.

1. THE PROBLEM

The one-criterial problem (Wagner 1969) can be introduced as follows: The process of production and stock-holding for one item and T periods is considered. The production figures $x_t \geq 0$ for t^{th} period, $t = 1, 2, \ldots, T$, have to be chosen such that given deterministic demand $d_t \geq 0$ is satisfied for all t and that the total sum of set-up costs and holding costs is minimal. The fixed set-up costs arising if $x_t > 0$ are denoted by $c > 0$ and the per-unit period linear holding costs are denoted by $h > 0$. If the stock at the end of t^{th} period is denoted by y_t the demand is satisfied in the case that $y_t = y_{t-1} + x_t - d_t$ is non-negative. Usually the assumption is made that the stock equals zero at the beginning and at the end of the considered planning period $1, 2, \ldots, T$, i.e.

$$y_0 = y_T = 0.$$

Then the model can be described by the formulae (1)–(4):

$$y_t = y_{t-1} + x_t - d_t \tag{1}$$

$t = 1, 2, \ldots, T,$

$$x_t \geq 0, \; y_t \geq 0, \tag{2}$$

$$y_0 = y_T = 0, \tag{3}$$

$$F_1 = c \sum_{t=1}^{T} \operatorname{sign} x_t + h \sum_{t=1}^{T} y_t \rightarrow \min \tag{4}$$

The second criterion consisting in the minimization of the stock is provided by formula (5):

$$F_2 = \sum_{t=1}^{T} y_t \rightarrow \min \tag{5}$$

The objective F_2 makes sense only if the first criterion is also of interest, since the solution $x = \{x_t = d_t\} \sum_{t=1}^{T}$ provides always $F_2 = 0$.

The one-criterial problem will be denoted by $M(c, h)$. Feasible solutions of this model have to satisfy conditions (1)–(3), while optimal solutions are feasible solutions minimizing the function (4).

Let two feasible solutions be considered which have the values F'_1, F'_2 and F''_1, F''_2, respectively. The first solution is said to be dominated by the second one if $F'_1 \geq F''_1$ and $F'_2 \geq F''_2$. It is strongly dominated if the inequalities hold and one of them is strong. An efficient solution can be defined as a feasible solution which is not strongly dominated by any other feasible solution. In other words, the feasible solution associated with the values F'_1, F'_2 is efficient if it follows for any other feasible solution that

$F''_1 < F'_1$ implies $F'_2 < F''_2$ and

$F''_2 < F'_2$ implies $F'_1 < F''_1$.

The main task in multi-criterial optimization is to find set EFF of all efficient solutions of a given