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STABILITY OF A TWO-STAGE PRODUCTION AND INVENTORY MODEL

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ABSTRACT

The serial assembly model is considered. The problem is formulated to find sets of cost inputs for which solutions found by a recursion procedure remain valid. For simplicity a solution of this problem is provided for the two-stage problem. The paper shows that the stability region of cost inputs forms a convex cone in $\mathbb{R}^4$ and consists of a system of linear inequalities. An algorithm is provided to compute the parameters of this cone and several cases of changing only two parameters are displayed graphically.

1. THE MULTI-STAGE SERIAL ASSEMBLY PROBLEM

This well-known model has been studied by many authors (Zangwill [1], Love [2], Lambrecht et al. [3], Graves [4], Blackburn and Millen [5], Chand [6], Afentakis et al. [7]). A comprehensive formulation of the problem is given below:

$$\min \sum_{s=1}^{S} \sum_{t=1}^{T} (c_s \operatorname{sign} x_{st} + h_s I_{st})$$

Subject to:

$$\begin{aligned}
&I_{st} = I_{s,t-1} + x_{st} - x_{s+1,t} && s=1,...,S,\ t=1,...,T \\
&x_{st} \geqslant 0, \quad I_{st} \geqslant 0 && s=1,...,S,\ t=1,...,T \qquad (P) \\
&I_{s0} = I_{sT} = 0 && s=1,...,S \\
&x_{s+1,t} = d_t && t=1,...,T
\end{aligned}$$

This model has the following economic interpretation: A production–inventory system consists of S facilities in series. The input to facility $s+1$ comes from facility s. Facility S produces assemblies which are used to supply the customer demand d_t for periods $t=1,...,T$. All facilities may carry inventories. It is assumed that production and shipment are instantaneous and that one unit of production at facility $s+1$ requires one unit of input from facility s. Backlogging of demand is not allowed. Let x_{st} denote the production at facility s in period t, and the cost of production be provided by $c_s$sign x_{st}. The stock at the end of t-th period at facility s is denoted by I_{st}. The holding cost is provide by $h_s I_{st}$. It is easy to see that the model has been designed to determine a minimum cost production and inventory strategy for S facilities and T periods which meets the demand of all periods.

It is also well known that there are optimal solutions for this type of model which fulfil

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