

A Parametric Analysis of the Dynamic Lot-sizing Problem¹⁾

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Abstract: This paper gives the analysis of the parametrized dynamic lot-size model. Three cases of parametrization are studied here: (i) setup and holding costs, (ii) demand vector, (iii) costs and demand. For all these cases the stability region of the parameters is found, i.e. it is shown for which parameters a solution generated by *Wagner-Whitin's* algorithm remains valid. In the parametric analysis of the cases (i) and (ii) the parameter intervals are found by studying a simple system of inequalities. In the more complicated case (iii) the stability region of the two parameters is drawn by a computer program.

1. Introduction

In the dynamic lot-sizing problem (DLSP) the demand for the finished product occurs periodically and is known for T time periods in advance. Our models rely upon the assumption of linear inventory holding costs rather than the more general concave holding costs and holding costs remain constant in all time periods.

The DLSP is one of the best known standard model in OR/MS and the basic model of DLSP has been developed into many directions. Although the dynamic programming algorithm given by *Wagner* and *Whitin* (1958) for solving the uncapacitated DLSP can be considered as an effective one, heuristics are also studied frequently ([9], [6], [3]).

The theory of the multilevel lot-sizing problems is a useful generalization of the single-level DLSP, too ([5], [11], *Love* (1972), [4], [2], [1]) and relatively less effort has been devoted to the parametrical problems ([7], [8]).

The purpose of this paper is the investigation of the stability of an optimal schedule. Section 2 gives the parametric analysis of the objective function, while Section 3 gives that of the demand vector. Section 4 describes the two-parametric case.

2. Parametric analysis of the objective function

The DLSP is studied as

$$\sum_{t=1}^T (s \cdot \text{sign}(x_t) + h \cdot y_t) \rightarrow \min; \quad (1)$$

$$y_{t-1} + x_t = y_t + d_t, \quad t = 1, 2, \dots, T, \quad (2)$$

$$x_t \geq 0; \quad y_t \geq 0; \quad t = 1, 2, \dots, T, \quad (3)$$

$$y_0 = y_T = 0. \quad (4)$$

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