Utility versus Income-Based Altruism

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Abstract

In Dictator Game experiments where the information status of the participants varies we find that a certain type of proposer tends to reduce his offers when the recipient has incomplete information about the pie size. We also find that a certain type of recipient tends to reject too small offers in the Impunity Game when the proposer has incomplete information about the recipient type. To explain these puzzling results we reconsider Becker’s [1974] theory of altruism, which assumes that externalities are caused by other people’s utility. When incomplete information about the other person is introduced, it turns out that his approach predicts – in contrast to other theories of altruism - that some altruistic persons will change their behavior as observed in our experiments. Thus, a kind of utility based altruism (and spite as its opposite form) can be assumed as the main principle governing behavior in this class of games.

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1 Introduction

Experimental economists agree on one point: the narrow self-interested individual utility function of payoff maximization accurately describes the behavior of human beings only in some occasions. When it comes to describing other, non-selfish behavior, the accord ends.

Initial suggestions that altruistic behavior alone could be a principle motive were rejected by many experiments involving the Prisoners’ Dilemma (cf. e.g. Bolle and Ockenfuels [1990]), and the Ultimatum Game (cf. e.g. Güth et al. [1996]). When decisions are embedded in a strategic setting, we observe choices different from altruistic (or egotistic) ones. On the one hand, it seems that in some games with more than one stage other motives (such as positive or negative reciprocity, induced envy or inequity aversion) override the altruistic feelings. On the other hand, altruism could be the baseline for cooperative behavior in other games with more than one stage, such as the Centipede Game (McKelvey and Palfrey [1992]).

This proves that altruism is an elusive concept, not only in theory (as Simon [1993] has highlighted) but also in experimental economics. One way to isolate altruistic motives from others is to conduct experiments using only one-stage games. The most prominent experiment of this type is the Dictator Game. In this game Person A, the dictator, is able to decide how to share a pie between himself and Person B. Since the recipient is not able to react to the decision of the dictator, the dictator can be influenced by the recipient (if at all) only implicitly. Thus, the anonymously played Dictator Game can be seen as one test for search altruism motives, since Person A is able to express his preferences with respect to his willingness to reduce his own level of consumption in favor of Person B.

The core results of Dictator Game experiments are that the average offer to the recipient varies between 20% and 25%, that approximately 1/3 of participants give (almost) nothing (less than 10%) while roughly two-thirds of the participants give more than 10% and up to 50% of a pie of a $10 size (see e.g. Forsythe et al. [1994] which found support in later experiments by Hoffman et al. [1994], Camerer und Thaler [1995], Bolton and Zwick [1995], Eckel and Grossman [1996], Andreoni and Vesterlund [2002] and Andreoni and Vesterlund [2001]).

However, the behavior noted in the Dictator Game is not unanimously interpreted. An altruistic interpretation of these non-selfish choices is favored by Forsythe et al. [1994], Eckel and Grossman [1996], Andreoni and Vesterlund [2001], and others. Hoffman et al. [1994, 1996] were the first to doubt that such a high share of participants may have altruistic motives. To make sure that dictators were isolated in a way that there was no implicit social interaction, they conducted Dictator Game experiments with a double blind procedure. They (and later Johannesson and Persson [2000]) in a similar experiment found that not two-thirds of the participants had made choices according to certain social norms.²

A different kind of objection to the experiments of Hoffman et al. [1994] was raised by Bolton and Zwick [1995] and Bolton et al. [1998]. They argued that in the experiment of Hoffman et al. [1994] the decreased willingness of the dictator to share the pie with the recipient did not result from the increased anonymity but from the differences in the written instructions (compared to the baseline experiment of Forsythe et al. [1994]). Bolton et al. [1998] further concluded that participants, when making their choices as dictators, “act first to secure what they consider to be their own fair share.” Dictators do not behave in an altruistic but “in a self-interested manner” when they secure their own fair share.

In this paper we aim to offer a different perspective. While previous research on the Dictator Game was mainly concerned with the subject-experimenter relation, we refocus attention on the donator-recipient relation. We will use the typical Dictator Game (as determined by Forsythe et al. [1994]) as a benchmark to further investigate the decisive variables of altruistic behavior, in particular why and how dictators, when deciding about their offer, might be implicitly influenced by the recipients even if the game is played in complete anonymity.

Basically there are two different approaches to altruistic behavior, where the donator either aims to increase the income (cf. e.g. Collard [1978] and most other researchers) or the utility of the recipient (almost exclusively used by Becker [1974]). While in Section 2 of this paper we argue that under complete information it is difficult to discriminate between the two approaches, Section 3 reveals that they have different implications in Dictator Games with incomplete information.

In Section 4 the theoretical findings will be confronted with the result of an experiment inspired by Güth and Huck [1997]. In order to test the predictions of the utility-based approach, we conducted two Dictator Games: one under complete, one under incomplete information. In the latter experiment, the dictator knows the size of the pie while the recipient knows only the probability distribution of the potential pie sizes. Those dictators who received large pies either offered the recipient a share relatively close to the equal split of the large pie (and, thus, revealed the true size of their pie) or they pretended to have received a small pie and offered the recipient half of the small pie or even less. This specific behavior can be explained only by altruism as deduced from the utility-based approach.

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² For an approach based on these experimental results, cf. Levine [1998] to which we will refer below.

³ In reply to this experiment, Frohlich et al. [2001] suggested that in the increased anonymity setting the participants may have doubted that a recipient actually exists and therefore decreased the share transferred to their anonymous partner. For further contributions to this discussion cf. Bohnet and Frey [1999], Hoffman et al. [1999].
Section 5 is devoted to a second test of this approach. Bolton and Zwick [1995] modified the Dictator Game where the recipient could reject the amount he received by the dictator, whereas the dictator could keep his own share. This game, the 'Impunity Game', transforms the Dictator Game into a strategic game under incomplete information, in particular for the dictator. We conducted the Impunity Game and compared its results with the standard Dictator Game. We found out that, firstly, recipients were ready to decline positive offers if these were "too small", and that, secondly, those Dictators who anticipated potential rejections either decreased their transfer to zero or increased it above the amount where they expected a rejection. This behavior is explained again by a utility-based approach.

2 Altruism under Complete Information

In terms of utility theory, altruistic behavior is caused by external effects of "consumption". There are, as mentioned in section I, two competing approaches which differ in the origin of the externalities. In most approaches on altruism it is assumed that a person's utility is influenced by other persons' consumption of goods or by other persons' income.

\[ U_i = \sum_{j=1}^{n} x_j, \quad i \neq j \]

where \( x_i \) is consumption, and \( x_j \) represent the consumption of individuals \( j \) with whom altruist \( i \) interacts. Given (1), if \( j \)'s utility is increased if \( j \) enjoys a higher income.

Gary Becker [1974] has proposed a different utility-based setting:

\[ U_i = U(\sum_{j=1}^{n} U_j), \quad i=1,...,n \]

where \( U_i \) represents the utilities of individuals \( j \) with whom altruist \( i \) interacts. \( i \)'s utility is increased if \( j \)'s utility is increased or \( i \) is 'happy' if \( j \) is 'happy'.

Both interpretations rely on positive derivatives, i.e. \( \partial U/\partial x > 0 \) or \( \partial U/\partial U > 0 \). However, negative derivatives are possible, as well, where \( i \) is spiteful towards \( j \) so that \( i \)'s utility is increased when \( j \) faces a lower income or a lower utility, respectively.

The income-based utility approach is mostly preferred for reasons of tractability. Equation (2) constitutes a system of equations which may be solved for \( U_j \) and then results in (1). As for (2), the utilities \( U_i \) are required to be somewhere between cardinal and absolute values (cf. Bolle [1985]); thus, it seems more appealing to start with (1). "Solving" the equations means that "equilibrium" values \( U_i^* \) are determined where all persons have consistent expectations about the utilities of all others – a point which, in the next section, will prove to be of crucial importance when using the utility-based approach under incomplete information.

The first reason to support one of the two approaches can be deduced from the analysis of the emotional ties between persons. (2) describes the direct emotional link between persons (such as love or spite) depending on the sign of \( \partial U/\partial U > 0 \). (1) expresses the aggregation of all direct and indirect influences which may be described as altruism or malice, depending on the sign of \( \partial U/\partial x > 0 \).

The differing focus has consequences for indirect relationships. Under (2), \( i \) will help the friend of his friend (the enemy of his enemy) even if there is no direct relation to him. Under (1), it is not possible to grasp indirect ties. Further, under (2), changes in one emotional relation between two members of a group will change all behavioral relationships between the relevant members. Under (1), again, no such consequence could be expected.

The implication that one changing emotional relation in a group may influence the behavior within the whole group, is supported by psychologists and sociologists (cf. e.g. Granovetter [1973]), as well. Nevertheless, there seem to be no hard facts and no experiment clearly discriminating between these two approaches. This holds, however, only under complete information which is implicitly assumed when (1) is derived from (2).

The description of altruism in (2), however, is too general to be useful for the Dictator and the Impunity Game, the main focus of the present paper. Therefore, we will introduce a specification of altruistic behavior, which is combined with recent research on equity theory and which fits into this class of two-person games. In this class of games, Person 1 is able to distribute a pie of size \( p \) between the two, with \( x_i \) being transferred to Person 2 and \( x_j \) being kept by himself (\( x_i + x_j = p \)). Consider the following utility functions:

\[ U_i = x_i + a \left( \frac{x_i}{p} - s_i \right) U_j \]

and

\[ U_j = x_j + b \left( \frac{x_j}{p} - s_i \right) U_i \]

where we assume for simplicity's sake that the individual's utility is linearly increased in its own consumption, and where \( a \) and \( b \) are constant parameters of the two individuals. Based on recent research we further suppose that individuals have altruistic motives as long as their...
actual share $x_i$ of the pie $p$ is higher than a certain minimum share of $s$.

By introducing the minimum fairness standard it also becomes possible to avoid an exogenous specification of the sign of $\partial U_i/\partial U_j$. The derivations will be determined within the system where person $i$'s emotions may change into spite with the result that he is better off when $U_j$, $j \neq i$, decreases.

Solving the System (3), (4) leads for Person $i$ to

$$U_i = U_i^* = \frac{x_i + a \left( \frac{x_i}{p} - s_i \right) x_j}{1 - ab \left( \frac{x_i}{p} - s_i \right) x_j}, \quad j \neq i$$

The graphical solution of (5) is given in Figure 1 for two persons. The result of (5) corresponds to the outcome under the income-based approach on altruism.

- insert Figure 1 about here –

3 Altruism Under Incomplete Information

While in the usual Dictator Game both parties know the pie size, the main focus of the present paper is on a setting where the information status of the recipient and of the proposer is varied. In this section, we will focus on the kind of behavior the two altruism concepts predict when the recipient has incomplete information about the pie size.

One main feature distinguishing concepts on altruistic behavior from similar concepts is that altruistically-behaving individuals do explicitly consider the consequences of their choices (in the Dictator Game and elsewhere) for the recipient, while, for example, in fairness concepts (as proposed by Bolton et al. [1998]), Dictators are assumed to consider only the consequences of their choices for themselves. Thus, on the one hand, altruistic concepts allow a more thorough analysis of human behavior. On the other hand, the fact that a person considers the consequences of his choices increases the analytical requirements. In particular, we need to consider what expectations each individual forms about the state of his fellow persons. Therefore, as a first step in analyzing the incomplete information approach, it is necessary to explain the formation of consistent expectations.

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5 Incomplete information in Approaches of (1) constitutes no problem. It is possible to consider $E_i V_i$, i.e. $E_i$ expected value of $V_i$ (from the viewpoint of $i$ – therefore $E_i$), as the decisive utility. At the same time $V_i$ is assumed to measure $i$’s risk aversion, as well.

6 The general problem of coping with incomplete information in the utility-based approach requires a separate model. This paper merely aims to show that such an endeavor is worthwhile as this approach is capable of explaining phenomena which cannot be captured by other approaches.

11 Person 2 knows the distributions of these values. In
Sequential Equilibria (cf. Kreps and Wilson [1982]). Consistent means in both cases that expectations have certain similarities to consistent beliefs in Dictator Games. For this class of games, it is assumed now that the dictator does not know expectations to the specification of (3) the utility function which we will use for all kind of values as, for example, $E_x$. On the one hand, the consistent expectations $E_i U_j$ are solutions of a system of equations and, thus, different from expectation values of a certain distribution. On the other hand, under incomplete information, the two concepts have similarities. The requirement that Person $i$ takes into account his knowledge about Person $j$’s utility, i.e. (8) and (9) in the above example, makes $U_i$ a random variable (from $j$’s point of view) and $E_i U_j$ also a usual expectation value.}

3.2 A Simple Model of Utility-Based Altruism Under Incomplete Information

Having introduced consistent expectations under incomplete information we may apply these expectations to the specification of (3) the utility function which we will use for all kind of Dictator Games. For this class of games, it is assumed now that the dictator does not know $s_j$, the recipient’s standard of justice. The recipient does not know $p$ and $s_j$. Both have information about the respective distributions which describe the other’s incomplete information. $a$ and $b$ are assumed to be common knowledge.

In order to apply our approach of consistent expectations to this class of games with the information status given above, a further factor complicating the utility based approach has to be mentioned: $j$ has to consider a random variable $zU$ consisting of the random variable $x$ (the distribution of which is common knowledge) and $U_i$ about which $j$ has to develop consistent expectations. In the line of our approach we therefore assume that $j$ has to develop consistent expectations about $U_i$ and about $zU_i$. Instead of (4), the recipient’s utility under incomplete information is now described as

$$U_j = x_j + bE_j\left[ \frac{x_j}{p} - s_j \right] U_i$$

$$= x_j + bx_j E_2 U_2 \frac{U_1}{p} - bs_j E_2 U_1.$$  

Based on (3), the dictator’s utility under incomplete information is described by

$$U_i = x_i + a\left[ \frac{x_i}{p} - s_i \right] E_i U_j.$$  

In this specification, the consistent expectation $E_i U_j$, $E_i \frac{U_1}{p}$, and $E_i U_2$ are also conditional expectations, namely under the condition that person 1 has transferred $x_i$ to person 2.

Considering (17) from the dictator’s point of view leads to

$$E_i U_2 = x_i + bs_i E_i \frac{U_1}{p} - bE_i s_j E_i U_j.$$  

Regarding (18) from the viewpoint of the recipient yields

$$E_i U_j = E_j \left( p - x_j \right) + a\left( E_j \frac{p - x_j}{p} - E_j s_j \right) E_i U_j.$$  

Finally, we divide (18) by $p$ and then form the expectation from the recipient’s viewpoint:

$$E_i \frac{U_1}{p} = E_i \left[ 1 - \frac{x_i}{p} \right] + a\left( E_i \frac{p - x_i}{p} - E_i \frac{s_i}{p} \right) E_i U_j.$$
(18), (19), (20) is solved for $E_1 U_2$, $E_2 U_1$, $E_2 \frac{U_1}{p}$ since all other values are common knowledge. Inserting these values in (16) and (17) reveals the utilities which recipient and dictator enjoy. Since only the dictator makes a decision we focus on his utility:

\[
U_1 = x_1 + \alpha \left( \frac{x_1}{p} - s_1 \right) E_1 U_2 = \frac{x_2 + b E_1 s_2 (x_2 - E_2 p) + b x_2 \left( 1 - x_2 E_2 \frac{1}{p} \right)}{1 + a b s_2 \left( 1 - x_2 E_2 \frac{1}{p} - E_2 s_1 \right) - a b x_2 \left( E_2 \frac{1}{p} - x_2 E_2 \frac{1}{p^2} - E_2 \frac{s_1}{p} \right)},
\]

where $E_2$ denotes the recipient’s conditional expectation having received $x_2$. Under certainty, (21) is equal to (8). The dictator chooses $x_2$ such that (21) is maximized. A numerical example showing that there is a solution as described in this section is given in Appendix B.

## 4 A Dictator Experiment with an Uninformed Recipient

Having shown the implications of incomplete information for the utility-based approach on altruism we turn now to the experiment which focuses on the Dictator Game where the information status of the recipient is varied according to the manner described above. In the usual Dictator Game under complete information, Person 1 (the dictator) is endowed with a known amount of money which he can divide arbitrarily between himself and Person 2 (the recipient). In the present experiment the Dictator Game is varied insofar as the dictator is endowed with an amount $p$ which is not known to the recipient. She only knows that $p = p_s$, with probability $\alpha$ and $p = p_s$, with $1 - \alpha$, and she knows $p_s < p_s$.\(^{13}\)

### 4.1 Analysis

Under the income-based approach – whatever the specification of (1) is – the dictator is expected to offer the recipient the same amount $x_2$ and keep $x_1 = p - x_2$ for himself, irrespective of the recipient’s information status about the pie size $p$.

If the utility-based approach is applied, the dictator is expected to care about the recipient’s beliefs on $x_1$ and about her utility parameters. (18) implies that the dictator wants $U_2$ to be as large as possible, provided he himself gets a share $x_1/p > s_1$. (17) implies that this goal requires the recipient to assume $p$ to be small. (For a numerical example see Appendix B)

Experience from previous Dictator Game experiments implies that the recipient has expectations about the transfer $x_2$ she would receive under complete information. It is reasonable to assume that she expects no dictator to give more than half of his endowment. By giving more than $p_s/2$ the dictator would uncover that he has $p = p_s$. By proposing less than $p_s/2$ he can make the recipient believe that $p = p_s$, as we will see below, at least with a probability $\alpha' \geq \alpha$.

Given that the distribution of $p_s$, $s_1$, and $s_2$ are common knowledge, it is plausible to expect the dictators, if endowed with a large pie $p_s$, to decide as follows: First, there may be a fraction $\beta$ of dictators who will transfer more than $p_s/2$, indicating that they were endowed with $p_s$.

Since they are ready to reveal their type, they will give the same amount as under complete information. It makes sense to assume (supported by the subsequent analysis) that the $\beta$-fraction of dictators are those types who give most under complete information. Of course, $\beta$ is exogenously given but determined by the analysis of the situation.

Second, there is a fraction $1 - \beta$ of dictators who received a large pie and would have proposed not much more than $p_s/2$ under complete information. This fraction may increase his utility by reducing $x_s$ below $p_s/2$. Since we are more interested into the qualitative reasoning, we will make the subsequent plausibility analysis (for the exact updating process see Appendix C) by using (16) and (17): If dictators anticipate that the recipient’s utility can be increased by making her believe (with a certain probability) that there is only a small pie, they may decide to give less than $p_s/2$. In (16) this would mean that a high reduction of $E_2 \frac{U_1}{p}$ could be induced by a small reduction of $x_2$ (if $U_1$ is about the same from the recipients point of view). Hence, $U_1$ and, consequently, $U_2$ will be increased. On the other hand, $x_1$ and $x_2$ do not correspond anymore to their optimal values under complete information.

This makes it possible to distinguish between those who reveal the true size of their pie and those who hide behind the small pie. Those dictators (who would have given more than half of the small pie in the complete information setting) will now have to compare the ‘direct’ utility decrease from reducing their transfer to an amount less than half the small pie with the ‘direct’ utility increase from reducing their transfer below the optimal level. It is reasonable to expect that the ‘direct’ utility decrease is higher the more $x_1$ has to be reduced under incomplete information in comparison to the optimal $x_1$ under complete information.

This reasoning, however, still leaves open how much dictators pretending to have received the small pie are expected to propose. Under complete information, it would be optimal to offer $x_1 > p_s/2$, under incomplete information the dictator would like to reduce his proposal
as little as possible. According to the analysis he would prefer to transfer an amount exactly equal to $p_2/2$ or just below $p_2/2$. Yet, if the recipient, in her updating process of the distribution of $p$ does not only note whether $x_2 \leq p_2/2$, but also takes into account the exact amount of $x_2$, then $x_2 = p_2/2$ will make her "distrustful". Therefore, an extended analysis with a more sophisticated updating process should show that not necessarily all types of dictators would propose an amount of approximately $x_2 = p_2/2$, but that every type may have a personal optimum. Of course, the dictator can hide his large pie only if there are dictators who would offer the same amount $x_2$ when endowed with a small pie.

Those dictators with $p_2$ who are also among the "$1-\beta$ fraction" and who would offer $x_2 < p_2/2$ even under complete information will, since $E_p$ is reduced, further reduce $x_2$, as well. In contrast to this, dictators who are endowed with a small amount $p_2$ will suffer from the incomplete information situation and from the updating procedure of the recipients. It can be expected that they will increase $x_2$ (compared to the complete information situation) to compensate for the increased $\tilde{x}_2$.

**Conclusion 1:** In comparing Dictator Game experiments with complete and incomplete information, the following expectations can be deduced: The *utility-based approach* predicts for dictators endowed with $p_2$ three different behavioral patterns. First, there are dictators who offer a relatively large share (up to the equal split) under complete information and who are expected not to change behavior under incomplete information. The second type are those dictators who give not much more than $p_2/2$ and the third type those who give less than $p_2/2$ under complete information. Under incomplete information the second type will propose less (but close to) $p_2/2$, and the third type will further reduce their offer (partly to zero). (See Figure 2, bold curve.) For more differentiated types of dictators the described monotone matching of types and contributions may be less strict. The expected behavioral changes should show the same, but less extreme tendency. Furthermore, for a dictator endowed with $p_2$, the utility-based approach predicts that offers are increased.

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**4.2. Experimental Design and Procedure**

**DESIGN:** The present experiment encompassed two different treatments, the basic Dictator Game (Game 1) and the Dictator Game under asymmetric information (Game 2). The basic Dictator Game aimed to confirm previous results and to serve as a baseline treatment for comparison with Game 2. In both treatments, a dictator was anonymously matched with a recipient. The dictator received an endowment of either $p_2=10$ Euro or $p_2=1.15$ Euro.\(^{14}\) In

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\(^{14}\) The experiment was conducted in November 2001 using German currency. Thus, the pie size was then 19.55 DM for the large and 2.25 DM for the small pie. Therefore, no prominence effects could occur in the experiment.

Game 1 the recipient knew the endowment of the dictator. Game 2 differed from Game 1 only in one variable: the recipient did not know the exact size of the endowment but was informed that the dictator received a large pie of 10 Euro with probability $\alpha=5/8$ and a small pie of 1.15 Euro with probability $\beta$. All other variables were kept constant in both games (for the Instructions see Appendix A).

**ORGANIZATION:** 240 undergraduates from our University participated in this part of the experiment - 120 in each session. They were recruited through announcements in lectures. Participation required appearance at a prearranged place and time and was restricted to one session. Upon arrival participants were randomly assigned to their roles as dictator (Person A) or recipient (Person B). In both treatments 40 dictators were endowed with the large and 20 with the small pie. Throughout the sessions participants were placed in separate rooms. All experiments were conducted once, after the participants had received written and verbal instructions about the setting. All participants were randomly and anonymously matched.

**PROTOCOL:** In each session all dictators received an envelope containing the written instructions and the amount of the pie, split into many coins enabling the dictator to propose any amount to the recipient he preferred. The written instructions used the same script for both treatments with the only modification that in Game 1 the recipient was informed about the size of the pie the dictator had received and that in Game 2 the recipient was informed about the probability distribution with which the dictator had received either one of the two pies. To ensure complete privacy for the decision, cubicles were offered. The dictators put the amount devoted to the recipient back into the envelope, put the envelope into a box where all proposals were collected and pocketed their own share of the pie. The box was then transferred to room B and randomly distributed to the recipients after two neutral persons had registered the amount in each envelope in a third room. Thus, it was not possible to attribute any individual action to individual subjects.

**REMARK ON THE ANALYSIS OF BEHAVIOR:** When the experiment was designed it became clear that the smaller the small pie was in the incomplete information setting the easier it became to discriminate between the behavior of those dictators in the two treatments who received the large pie. At the same time, the smaller the small pie, the less it became possible to discriminate between behavior of those dictators in the two treatments who received the small pie. Since we decided that the decisions of dictators who received the large pie are more important, we chose the small pie to be "very small" with 1.15 Euro. Accordingly, we will restrict the analysis to those dictators who received the large pie and will only make some remarks on the behavior of the dictators who received the small pie.
4.3. Predictions

Starting with the unique ‘egoistic’ equilibrium prediction, the dictator would make no positive offer to the recipient no matter how the recipient is informed about the pie size the dictator received. Application of the income-based approach on altruism is straightforward as well. It results in (H0), the distribution of the dictator proposals should be the same under both conditions, irrespective of the information status of the recipients.

Application of the utility-based approach on altruism leads to the following hypotheses given the parameters of small and large pie and given the theoretical results presented in Figure 2. In comparison to Game 1, among those who received the large pie in Game 2 (where the recipient has incomplete information), we expect that

H1) the average transfer \( x \) of the dictators will decrease.

H2) more subjects will offer nothing or less than 0.6 Euro (about half the small pie).

H3) less subjects will offer amounts between 0.6 Euro and 2.5 Euro (about one quarter of the large pie and the modal offer in Game1).

H4) about the same number of subjects will offer between 2.5 and 5 Euro.

With respect to the separation between those Dictators who reveal and those who do not reveal the true size of the pie we did not optimize \( x \). Instead we decided to separate those types of Dictators by arguing that subjects who transfer significantly more than the average type in the complete information game are also willing to reveal their true size in the incomplete information game by transferring the same amount. More specifically, we chose \( x=2.5 \) Euro as a crucial amount to distinguish between these two types.

4.4. Experimental Results

In a first step we compare the data of the present €10 Dictator Game under complete information with the $10 Dictator Game experiment of Forsythe et al. [1994, p. 366] - which served as a baseline treatment in previous studies. The distribution of proposals and the average payoff (22.3% in the Forsythe et al. and 20.4% in the present experiment) are similar.\(^{15}\) (No significant difference by the Mann-Whitney U test, \( p=0.2912 \)). This indicates that the behavior of the present ‘population’ is comparable with earlier observations.

The analysis of the present study focuses on the comparison of the dictator’s willingness to transfer a certain share of his pie to the recipient when the information status of the recipient is varied. Starting with hypothesis H1, the average offer, dictators who had received the €10 pie in Game 2 proposed on average 11.4% of the pie, half of what dictators offered under complete information in Game 1 (20.4%). (An overview of all offers in the two games is given in Table 1.) A Kolmogorov-Smirnov Test verifies in favor of H1 that the distribution of proposals is significantly lower in Game 1 than in Game 2 (\( p<0.01 \)). H0 can be rejected.

With respect to the three hypotheses H2 to H4, we have ordered in Figure 3 the results of Games 1 and 2 in a way that the hypotheses can be tested. Proceeding with H2 the share of participants who made proposals between nothing and half the small pie, we observe an increase from 23% to 60% in the asymmetric information game (in support of H2, Fisher’s probability test shows \( p=0.017 \)). This observation indicates that many participants tried to signal to the recipient that they had received the small instead of the large pie. Moreover, the high share of subjects giving even less than half the small pie supports our approach in two ways: not only subjects who had given less than half of the small pie in game 1 reduced their offers further to (almost) zero, but also participants who had given slightly more than \( x/2 \) reduced their offers to an individually calculated optimum which was not equal to \( x/2 \).

Figure 3 also provides answers to the two further hypotheses: In (H3) we hypothesized a sharp decrease of offers in the range between 0.6 and 2.5 Euro. Our data suggest this to be true since there were only 17% in Game 2 – as opposed to 47% in Game 1 – who offered an amount between 60 cents and 2.50 Euro (in support of H2, \( p=0.005 \)).

Coming to the final hypothesis (H4) it was asserted that dictators who transfer a relatively high share of the large pie under complete information would do the same thing under asymmetric information because this type of dictator is ready signal the size of his large pie. Since he is willing to sacrifice more than the average offer he expects the recipient to be content with his proposal. In Game 1 the share of dictators of this type was 30%, in Game 2 it was 23%, showing no significant difference (\( p=0.3 \)) between the settings.

A final remark should be made about the small pie under incomplete information. Due to the small size of the small pie we did not expect significant differences in behavior between persons in both settings and the experimental results showed that the decisions were indeed nearly the same. However, it is interesting to note that dictators endowed with a small pie in the incomplete information game transferred either nothing or less than € 0.6, confirming our argument in case of a large pie: It was necessary for dictators who were endowed with a large pie and who wanted to pretend having a small pie to give less than € 0.6.

Result 1: We observe significant changes in dictator behavior once the recipient has incomplete information. Only those dictators who offered more than average under complete information behave in the same way when the recipient does not know their pie size. They expect the increase of the recipient’s utility to be sufficient even if they reveal the size of their pie. Dictators making average offers or less in the complete information setting, reduced their offers to half of the small pie or even less under incomplete information.
5 The Impunity Game

In this section, we further test the two approaches by focusing on another variation of the Dictator Game, the Impunity Game (by Bolton and Zwick [1995]). In the Impunity Game the dictator is again endowed with known amount of money which he may divide between the recipient and himself. The recipient is given the choice to either accept or decline the dictator’s offer. In the latter case $x_2$ is lost and not given back to the dictator. The dictator, however, can keep his own share $x_1$, irrespective of the choice of the recipient.\footnote{Bolton and Zwick [1995] made a binary choice experiment which does not allow to analyze the recipient’s behavior in our sense. The same holds for the experiments of Gäthe and Huck [1997] who used the strategy method. Both designs do not fit in with the requirements of the present approach and in both experiments the dictators probably were not informed about the choice of the recipient. For a similar experiment close to the Impunity Game, see also Fehrler and Güth [2002].}

5.1 Analysis

Dictators, when deciding about $x_0$, expect to be confronted not only with a single type of recipient but with a distribution of types. Thus, they have to deal with incomplete information about the final income and the utility of their recipient. When dictators are informed about the rejection or acceptance of their proposal there is a clear difference between the income-based and the utility-based approach. Under the income-based approach, no recipient should reject any offer $x_0$. Under the utility-based approach, there might be recipients who reject small offers.\footnote{It is crucial for the analysis that dictators are informed about the recipient’s choice. Otherwise, if transfers are rejected without dictators being informed, additional motives must be considered.} In order to make this plausible, we use the utilities of (16) and (17).

Let us assume that there is a fraction $\beta(x_0)$ of recipients who accept the offer $x_0$, and let $\bar{E}(x_0)$ be the conditional expectation of the dictator about $s_0$ based on those recipients who do accept the offer $x_0$. $\bar{E}(x_0)$ is the conditional expectation based on those recipients who reject the offer. $\bar{E}(x_0)$ describes the recipient’s conditional expectation after receiving $x_0$. It is a plausible assumption that $\beta$, $\bar{E}$, and $s_0$ are increasing functions of $x_0$ while $\bar{E}(x_0)$ is decreasing. If the dictator uses a pure monotone strategy the recipient is able to determine $s_0$ exactly from the transfer $x_0$, i.e. $\bar{E}(x_0) = s_0$. Under these conditions we look for consistent expectation values $E_iU_2$ and $E_2U_i$ determining the utilities of the dictator and the recipient.

\begin{align}
U_1 &= x_1 + a \left( \frac{x_1}{p} - s_1 \right) E_1U_2 \\
U_2 &= x_2 + b \left( \frac{x_2}{p} - s_2 \right) E_2U_i
\end{align}

with $x_1 = x_2$ or $x_2 = 0$ (if the recipient rejects the offer).

\footnote{16 Bolton and Zwick [1995] made a binary choice experiment which does not allow to analyze the recipient’s behavior in our sense. The same holds for the experiments of Gäthe and Huck [1997] who used the strategy method. Both designs do not fit in with the requirements of the present approach and in both experiments the dictators probably were not informed about the choice of the recipient. For a similar experiment close to the Impunity Game, see also Fehrler and Güth [2002].}
recipient, the dictator had to put the note with his pseudonym into the envelope, as well. The envelopes were collected and distributed in the same way as in the first part of the experiment. Recipients either collected the transfer and chose on the note “offer accepted” or rejected the transfer (left the money in the envelope – if any was inside) and stated “offer rejected”. Then all envelopes were put into the box by the recipients and the box was brought to the room of the dictators. All decisions were read aloud to the dictators stating the pseudonym of each dictator and the decision of the recipient.

5.3 Predictions

The income-based approach on altruism expects that in the Impunity Game the dictators again make the same proposals as in the Dictator Game and that recipients accept all positive transfers (H0`). The utility-based approach expects that

H5) recipients will reject positive offers up to a certain amount.

Section 5.1 also allows for the further hypotheses that - compared to Game 1 - in Game 3

H6) a higher share of zero transfers, and
H7) a lower share of small transfers,
H8) a higher share of high transfers will appear.

5.4 Experimental Results

In the Impunity Game, we observed an average transfer of 22.2%, virtually the same amount as in the standard Dictator Games of e.g. Forsythe [1994] or of the present experiment, while the standard deviation was 1.63 in the standard Dictator Game and 3.34 in the Impunity Game. To find out whether the distribution of payoffs are different we run a Levene Test for equality of variances which showed that the variances of offers were different in the two games (p=0.038) showing first support for H6 to H8.

However, let us begin with hypothesis 5. Out of the 50 offers in Game 3, there were in total 6 rejections (significantly different from zero, p=0.013). Recipients rejected all (five) positive offers less than 10% of the pie size – i.e. of less than 1 Euro and there was one rejection on a 1 Euro offer. This shows that recipients did not reckon with the crumb of a pie.

With respect to hypotheses H6 to H8 we present the list of all offers in Table 1. According to our hypotheses, we subdivided (see Figure 4) these observations into four classes of offers, no offer, offers of less than 2 Euro, offers between 2 Euro and 4.5 Euro and offers of up to 5 Euro. As Figure 4 shows, in support of H6, 26% of the participants decided to make no transfer at all, significantly more than in our standard Dictator Game (p=0.046). In support of H7 (p=0.01) offers of less than 2 Euro where only observed in 16% of the cases while in the standard Dictator Game most offers were in this class (42.5%). With respect to the last hypothesis (H8), we conjecture that all participants who aimed to make sure that the recipient will be ‘happy’ with the transfer put ‘some more’ money into the envelope, with the result that in the medium range of transfers (between 2 Euro and 4.5 Euro) we observe merely the same frequency while at the high end of transfers (around 5 Euro) there were significantly more transfers in the Impunity Game (p=0.06) than in the standard Dictator Game.

Result 2: We observe significant changes in dictator behavior once the recipient is able to reject his own share of the pie (which the dictator has transferred to him). Only a small share of Dictators make positive transfers of less than 20% in the Impunity Game and most of these transfers are rejected. The majority of dictators either decide to keep all the pie or to offer sufficiently higher amounts to make sure that the recipient is satisfied with his share.

6 Summary

The present experiments compared the willingness of dictators to make offers to anonymous recipients when the information status of the dictator or of the recipients was varied. In the baseline treatment where both were fully informed, dictators gave similar amounts as in previous studies. In the second treatment where the recipient was only informed about the probability distribution of the pie sizes, dictators still gave non-trivial amounts, but some of them significantly reduced their transfers. Using the complete information treatment we differentiated between three types of dictators: Dictators who keep the complete pie for themselves, dictators who transfer less than or the average offer and dictators who offer more than average (up to the equal split of the pie). Having received the large pie, the first and the third type of Dictators did not change their behavior under incomplete information. The second type, however, preferred to hide their true endowment by reducing the offer: They induced the recipient to believe that he had received a considerable amount of the pie instead of a small amount of the large pie.

This type of dictator did so for good reasons, since he aims to make the recipient ‘happy’ by signalling generosity. As our experiment on the Impunity Game revealed, some recipients were indeed ‘unhappy’ about the suggested split of the pie. Most positive offers of less than 20% were rejected. Using the same typing of dictators, if the second type (who would have transferred less than average in the Dictator Game) anticipated rejections in the Impunity Game, he either refrained at all from trying to make the recipient happy and kept the whole pie or made sure that the recipient will feel happy by making higher transfers.

The utility-based approach on altruism (as suggested by Becker [1974]) is able to give a thorough explanation of the observed behavior. Since dictators may have anticipated that
recipients have a ‘spite’ component in their utility functions, in the Dictator Game with incomplete information they may propose offers which could be interpreted as considerable amounts of the pie, whatever its size. Thereby, dictators expect to increase the recipients’ utility (and, thus, their own utility) more than by revealing the true size of their pie. Yet, having determined the borderline between altruism and spite within the system, the utility-based approach is also consistent with the empirical evidence on the Impunity Game where the recipients can explicitly express the spite component in their utility function and where dictators either stop any transfer or increase the offer to an amount which can be regarded – again – as considerable in order to avoid any utility reduction by a rejection.

Our results have consequences for newer descriptive theories which aim to capture human behavior beyond the narrow self-interest. A major topic in the discussion of how to model this behavior was whether – as Bolton et al. [1998] formulated – dictators make offers “in order to improve the welfare of others” or to “secure what they consider to be their own fair share.” A main insight of our analysis is that dictators were not only interested in the fair share from their own point of view but that they also considered the recipient’s position: they explicitly decided in an altruistic way. This does not mean that equity-oriented approaches have no explanation. Rather, to the contrary, having introduced minimum standards of fairness as a means to determine the borderline between altruism and spite, it became possible to combine two approaches which have previously been used in a contradicting way. This approach on altruism (which traces back to Becker [1974]) is able to explain a wider range of behavior in the class of Dictator Games.18 Thus, we believe that our interpretation of the utility-based approach should be further tested in other social situations in and outside the lab where our approach places a lower bound of altruistic behavior.

References

Andreoni, J.; Miller, J.H.: Giving according to GARP: An Experimental Study of Rationality and Altruism, Econometrica 70, 737-753 (2002).


18 Levine [1998] e.g. explains behavior in the Centipede Game, in Public Goods Games, and in market games with a utility function similar to (10) which again may be based on Becker [1974].

19 A first example of similar behavior outside the lab is given, when the bequest of two children needs to be determined. Stark and Zhang [2002, p. 21] argue “Parents who are equally altruistic towards their children may consider leaving a larger bequest to the lower-earning child 2 (a “compensation” act). However, because the division of bequests is public information, unequal division is tantamount to a public statement that child 2’s earnings are relatively low—a declaration that can embarrass child 2.” This argument is in line with the utility-based approach on altruism.


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**Figure 1:** Altruism in Becker’s approach.

**Figure 2:** Expected changes of the distribution of offers from a dictator endowed with $p_x$. Extreme assumptions about types.
**Figure 3:** Cumulative Results of Dictator Offers in Game 1 and Game 2 (according to Hypotheses 2 through 4)

**Figure 4:** Cumulative Results of Dictator Offers in Game 1 and Game 3 (according to Hypotheses 6 through 8)

**Table 1:** Offers of Dictators in Game 1 (Dictator Game with complete information), Game 2 (Dictator Game with incomplete information) and Game 3 (Impunity Game)
Appendix A: Instructions to the players in the Dictator Experiment

In the description the instructions for player A are presented. Differences corresponding to the three treatments are indicated in boldface. For the instructions of Person B the obvious changes were made.

Instructions For Player A

You have been asked to participate in an economics experiment on individual decision making. For your participation you may earn some money which will be paid to you right away. Before you make any decision please read carefully the following instructions. If you have any questions, don’t hesitate to ask the experimenter.

In this experiment each of you will be paired with a different person who is in another room. This is room A and you are Person A. The person who will be paired with you is Person B in Room B. You will not be told who these people in Room B are, neither during nor after the experiment, and they will not be told who you in Room A are, neither during nor after the experiment. You will notice that there are other people in the same room with you who are also participating in the experiment. You will not be paired with any of these people. The decisions that they make will have absolutely no effect on you nor will any of your decisions affect them. The experiment is conducted as follows: A sum of DM 19.55 (DM 2.25) has been allocated to you in coins in the envelope.

Game 1 and Game 3: The person B who is matched with you knows that you have received this amount. You are now asked to propose how much of this each person is to receive. You are free to propose any amount you like to person B: nothing, something or the whole sum.

Game 3 (in addition): Person B in room B may accept or reject your proposal. If person B accepts your proposal, both of you will pocket the respective amount. If person B rejects your proposal, you will keep your own share of the pie and person B receives no payoff. At the end of the session you will be informed about the decision of B, by using your pseudonym.

Game 2: There are 39 (40) more players who have received DM 19.55 and 20 (19) more players who received DM 2.25. Person B who is paired with you does not know the exact amount allocated to you. Person B knows that you have received DM 2.25 with a probability of 33.3% and DM 19.55 with a prob. of 66.7%. You are asked to propose how much of the amount of DM 19.55 (DM 2.25) each person is to receive. You are free to propose any amount you like to person B: nothing, something or the complete sum.

For your decision you may use the cubicles in the room. You will have five minutes to come to a decision about your proposal. If you made your decision about the amount which you like to propose to person B, put the respective amount into the envelope and put the envelope into the box next to your cubicle.

Game 3 (in addition): Indicate your pseudonym on the second note and put the note into the envelope, as well.

Then you may pocket the amount you have allocated to yourself right away. Do not talk to the other people in your room until your session is completed. Do not be concerned if other people make their decision before you.

Appendix B: A Numerical Example of the Utility Based Approach in the Dictator game with an uninformed recipient

In this numerical example it is assumed that there is only one type, i.e. $x_1 = s_2 = x_2 = 0.4$, $a = b = 2$ and $p_0 = 1$, $p_1 = 10$ complete the assumptions.

Under complete information, a dictator would maximise $U_1$ from (8) which is equal to

$$U_1 = \frac{\frac{x_1}{p} + a \left( \frac{x_1}{p} \right) \frac{x_2}{p}}{1 - ab \left( \frac{x_1}{p} - \frac{x_2}{p} \right) \left( \frac{x_1}{p} - \frac{x_2}{p} \right)}.$$

He would like to keep about $\frac{x_1}{p} = \frac{2}{3}$ of the pie and would offer $\frac{1}{3}$ to the recipient. The dictator’s utility is then $\frac{U_1}{p} = 0.79$.

Under incomplete information the same dictator, if endowed with a large pie, may realize a higher utility if he pretends to have a small pie by offering $x_1 = p_1/3$. Because there is only one type ($s_1 = 0.4$) the dictator cannot offer any other amount without indicating that he is cheating. If $\alpha = 0.9$ then this strategy is profitable. The dictator earns (see (21)) $\frac{U_1}{p} = 0.92$.

Note that the recipient anticipates the dictator’s strategy and thus sets $\beta = 0$ and $\alpha' = \alpha$.

For small $\alpha$, for example $\alpha = 0.1$, such a strategy does no longer pay. On the other hand, dictators would not decide for a pure strategy $x_1 = p_1/3$ because, then, $\beta = 1$ implies $\alpha' = 1$, which would make cheating completely profitable. Instead, a fraction $\beta$ of dictators choose the strategy $x_1 = p_1/3$. This fraction is so large that the choice of $x_2 = p_1/3$ provides the dictators with the same utility as $x_2 = p_1/3$. For $\alpha = 0.1$, this fraction $\beta$ must be 0.95 which is plausible because of the small a priori probability $\alpha$. 

Appendix B: A Numerical Example of the Utility Based Approach in the Dictator game with an uninformed recipient
Appendix C: Probability Updating of the Recipient about the size of the pie in the Dictator Game with Incomplete Information

With respect to the fraction 1 - \( \beta \) of dictators who received a large pie and who may decide for a transfer of \( x_s \leq \frac{p_s}{2} \), it is assumed that the recipient, when 'calculating' her utility merely considers the fact that the amount is \( x_s \leq \frac{p_s}{2} \), i.e. she does not take into account the exact proposal of \( x_s \). In this case the recipient will update the probability that the dictator is endowed with a small pie by

\[
(25) \quad \alpha' = \frac{\alpha}{1 - \beta + a\beta}.
\]

Thus, by giving \( 0 \leq x_s \leq \frac{p_s}{2} \), the dictator makes the recipient expect

\[
(26) \quad E_2 p = \alpha' p_s + (1 - \alpha') p_L.
\]

\[
(27) \quad E_2 \frac{1}{p} = \alpha' \frac{1}{p_s} + (1 - \alpha') \frac{1}{p_L}
\]

\[
(28) \quad E_2 \frac{1}{p^2} = \alpha' \frac{1}{p_s^2} + (1 - \alpha') \frac{1}{p_L^2}
\]

\[
(29) \quad E_2 s_1 = \alpha' E[s_1 / \text{all types of dictators}] + (1 - \alpha') E[s_1 / \text{types who do not uncover } p_s]
\]

\[
(30) \quad E_2 \frac{s_1}{p} = \alpha' E[s_1 / \text{all types of dictators}] + (1 - \alpha') E[s_1 / \text{types who do not uncover } p_L]
\]

(25) - (30) may serve to compute \( U_1 \) for \( x_r = p_r/2 \). In order to determine the optimal decision of a dictator provided with \( p_r \) in a dictator/recipient relationship with the described simple updating process, we have to compare the value of \( U_1 \) under the reduced transfer with \( U_1 \) for the optimal \( x_r \) from (8).

Appendix D: On the subgame perfect equilibrium of the Impunity game

After the dictator has chosen \( x_r \) and the recipient \( x_s = 0 \) or \( x_s \) consistent expectations are formed. The recipient takes into account (22) and the dictator (23) leading

\[
(31) \quad E_1 U_1 = x_1 + a \left( \frac{x_1}{p} - \tilde{s}_1(x_2) \right) E_1 U_2
\]

\[
(32) \quad E_1 U_2 = \left[ x_2 + b \left( \frac{x_2}{p} - \tilde{s}_2(x_1) \right) E_1 U_1 \right]
\]

with

\[
(33) \quad \tilde{s}_1(x_1) =\begin{cases} \tilde{s}_1(x_1) & \text{if } x_r = x_1, \\ \tilde{s}_1(x_1) & \text{if } x_r = 0 \end{cases}
\]

Solving (31) and (32) for \( E_1 U_2 \) and \( E_1 U_1 \), we get

\[
(34) \quad E_1 U_2 = \frac{x_2 + b \left( \frac{x_2}{p} - \tilde{s}_2(x_2) \right) x_1}{1 - ab \left( \frac{x_2}{p} - \tilde{s}_2(x_2) \left( \frac{x_1}{p} - \tilde{s}_1(x_2) \right) \right)}
\]

\[
(35) \quad E_2 U_1 = \frac{x_1 + a \left( \frac{x_1}{p} - \tilde{s}_1(x_2) \right)}{1 - ab \left( \frac{x_2}{p} - \tilde{s}_2(x_2) \left( \frac{x_1}{p} - \tilde{s}_1(x_2) \right) \right)}
\]

Substituting \( E_2 U_1 \) in (23) by (35) leads to \( U_2 \).

The recipient has to choose between \( x_s = x_r \) or \( x_s = 0 \) depending on which decision leads to the higher utility. For given \( x_r \), \( \tilde{s}_1(x_r, \cdot) \), and \( \tilde{s}_2(x_r, \cdot) \), the decision depends on \( s_0 \). For large enough \( s_0 \), \( x_s = 0 \) and for small enough \( s_0 \), \( x_s = x_r \) are optimal. Thus, there is a crucial \( \tilde{s}_s \) which describes the borderline between acceptance and rejection of the offer (\( \tilde{s}_s = 1 \) is possible). However, each \( \tilde{s}_s \) determines \( \tilde{s}_1 \) and \( \tilde{s}_2 \) leading to a difficult computation of \( \tilde{s}_s(x_r) \). We have to find the \( \tilde{s}_s \) which, for given \( x_r \), equates \( U_2 (x_s = 0) \) and \( U_2 (x_s = x_r) \).
where $\bar{s}_2$ and $\hat{s}_2$ are determined as the expectation values under the condition that $s_2 > s_2^*$; or $s_2 < s_2^*$.

**Lemma 1:** Let $F(s_2)$ be a continuous distribution function on $[0,1]$. Then there is always at least one $s_2(x_2)$, connected with

$$\bar{s}_2(x_2) = \int_{s_2 \rightarrow 0}^* s_2 \, dF(s_2)$$

for $F(s_2) > 0$, $\bar{s}_2(x_2, F(s_2)) = 0 = 0$

(38) $\bar{s}_2(x_2) = \int_{s_2 \rightarrow 0}^* s_2 \, dF(s_2)$

so that it is optimal for all types of recipients $s_2 > s_2^*(x_2)$ to reject the offer and for all types of recipients $s_2 < s_2^*(x_2)$ to accept the offer.

**Proof:** All types with $s_2 < \frac{1}{p}$ have no incentive to reject the offer ($\frac{1}{p} > s_2$ provided) because both persons have altruistic feelings for each other. However, it does not pay for the recipient to reject the transfer, even if $s_2$ is slightly larger than $\frac{1}{p}$ because the second term in (23) is rather small so that the reduction of $x_2$ from $x_2$ to $0$ dominates.

Increasing $s_2^*$ from $\frac{1}{p}$ to larger values is accompanied by increasing $\bar{s}_2$ and increasing $\bar{s}_2 > \hat{s}_2$. Thus (35) implies that $E_z U_z$ decreases for $x_2 = x_2$ and even more for $x_2 = 0$ because $\frac{1}{p} - s_2$ has lower (negative) values. It may happen that $U_z(x_2 = x_2)$ is larger than $U_z(x_2 = 0)$ for all $s_2^* < 1$, leading to $s_2^*(x_2) = 1$, $\hat{s}_2(x_2) = 1$, $\bar{s}_2(x_2) =$ unconditional expectation.

On the other hand, having defined $\hat{s}_2$ and $\hat{s}_2$, by (36) and (37), when $s_2$ is increased, there might be an $s_2^*$ with $U_z(x_2 = x_2) = U_z(x_2 = 0)$. Then, for given $s_2^*$ and the corresponding value of $\hat{s}_2$ and $\hat{s}_2$, every type of recipient $s_2 < s_2^*$ is better off if she accepts the offer and every type $s_2 > s_2^*$ is better off if she rejects the offer.

**Summary:** there is at least one $s_2^*(x_2)$ with consistent conditional expectations $\bar{s}_2(x_2)$ and $\hat{s}_2(x_2)$ such that all types $s_2 > s_2^*(x_2)$ reject an offer $x_2$ and all types $s_2 < s_2^*(x_2)$ accept it.

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20 Let us assume a continuous distribution function of $s_2$ so that we need not determine behavior for $s_2 = s_2^*$.

---

**Lemma 2:** The "minimum solution" defined in Lemma 1 $s_2^*(x_2)$ of $U_z(x_2 = x_2) = U_z(x_2 = 0)$ increases with $x_2$.

**Proof:** If the equation is fulfilled for $x_2$, then $U_z(x_2 = x_2 + \varepsilon) > U_z(x_2 = 0)$.

**Lemma 3:** $s_2^*(x_2) \rightarrow 0$ for $x_2 \rightarrow 0$

**Proof:** Because of Lemma 2, $s_2^*(x_2)$ will converge to a certain value $s_2^*$ which is accompanied by $s_2 > \hat{s}_2$.

Thus (35) implies that $E_z U_z(x_2 = 0) \rightarrow \hat{\lambda} E_z U_z(x_2 = 0)$ with $\hat{\lambda} < 1$ for $x_2 \rightarrow 0$, and (23) implies that, for $x_2 \rightarrow 0$, the recipient is better off with $s_2^* = 0$.

For the sake of simplicity, let us assume that $s_2^*(x_2)$ is unique, i.e. the dictator is confronted with a unique function $s_2^*(x_2)$, and let us now turn to the dictator’s problem of determining $x_2$.

From $s_2^*(x_2)$ we get the probability of rejection of an offer $x_2$

(38) \[ \beta(x_2) = \int_{s_2 \rightarrow 0}^* dF(s_2) \]

Thus, the dictator’s ex ante expected utility from a transfer $x_2$ is

(39) \[ U_z^{\text{max}} = x_2 + 0 \left( \frac{1}{p} - s_2^* \right) \left[ \beta(x_2) + (1 - \beta(x_2)) E_z U_z(x_2 = 0) \right] \]

where $EU_z$ results from (34) with $x_2 = \hat{s}_2$ for $x_2 = 0$ and $s_2 = \bar{s}_2$ for $x_2 = x_2$.

As Lemma 3 reveals, for small enough $x_2$ nearly all recipients will reject the offer, i.e. $\beta = 1$.

Under these circumstances, the dictator is better off if he sets $x_1 = p$, i.e. if he keeps the complete amount for himself. Thus, small transfers are not optimal for the dictator; he should either increase them, or decrease them to 0.